Polynomials 6c - Classifying the Zeros of a Polynomial Functions

Standards: A-APR.2, A-APR.3, F-IF.7c, N-CN.9

Learning Target(s):

How many zeros does a polynomial have?

How can we find all the exact zeros given a graph?

(erase to show)

The Fundamental Theorem of Algebra

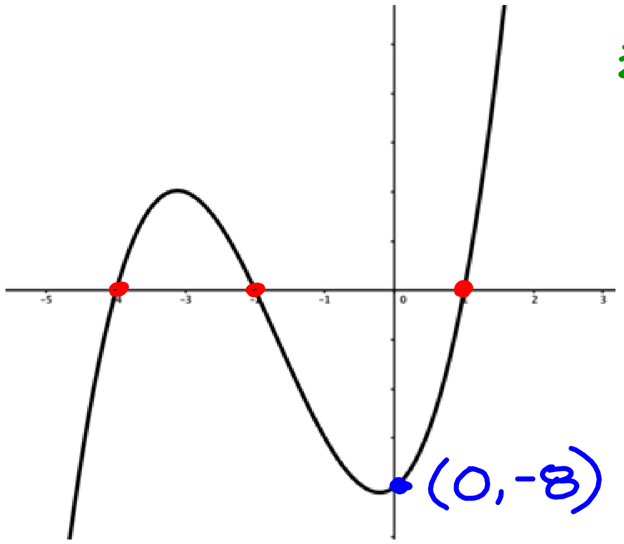
states that every nth degree polynomial has **exactly n zeros**.

- In some cases, a zero of a polynomial may be repeated. For example, the function $f(x)=(x+1)(x-3)(x-3)$ has three zeros: x=1, 3, & 3
  - \> In this case, we say, "f(x) has zeros at x=1 and x=3 (**multiplicity 2**)."
- This means that instead of crossing the x-axis at x=3, the graph of f(x) will have a "**point of tangency**" at x=3.
- If a polynomial has complex zeros, they will always happen in pairs.
  - \> If *a + bi* is a zero of the function, then *a - bi* will also be a zero of that function. These are called "complex conjugates."
  - \> For example, the function $g(x)=x^2-6x+13$ has two complex zeros: *3 + 2i* & *3 - 2i*.

* **Example 1:**

3 zeros

The graph of **P(x)** is shown below. **P(x)** is a 3rd degree polynomial (a "cubic" function). The scale used on the x-axis is 1 unit. The scale used on the y-axis is 2 units. Write the symbolic representation of **P(x)**.



zeros: -4, -2, 1

$(x+4)(x+2)(x-1)$

$(0+4)(0+2)(0-1)$

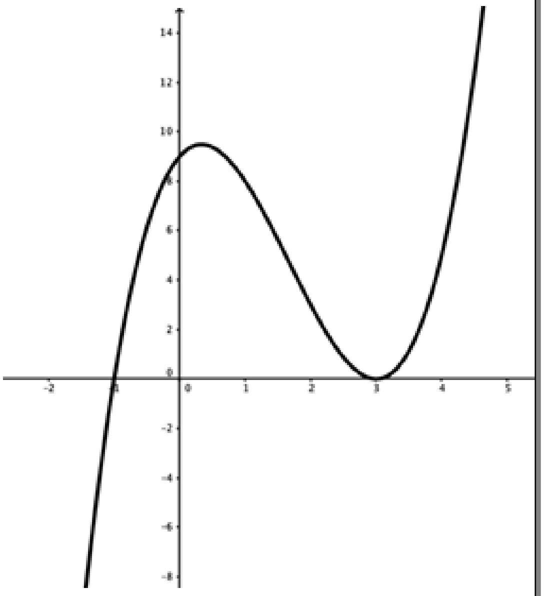
$(4)(2)(-1)$

(-8)

$$\boxed{P(x) = (x+4)(x+2)(x-1)}$$

Example 2: (HW)

The graph of **R(x)** is shown below. **R(x)** is a 3rd degree polynomial (a "cubic" function). The scale used on the x-axis is 1 unit. The scale used on the y-axis is 2 units. Write the symbolic representation of **R(x)**.



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* Example 3:

4 zeros

The graph of **C(x)** is shown below. **C(x)** is a 4th degree polynomial (a "quartic" function). The scale used on the x-axis is 1 unit. The scale used on the y-axis is 2 units. Write the symbolic representation of **C(x)**.

zeros: -1, 2, 4

bounce

$(x+1)(x-2)^2(x-4)$

$(0+1)(0-2)^2(0-4)$

$(1)(-2)^2(-4)$

$(1)(4)(-4)$

$(-16) \rightarrow 16$

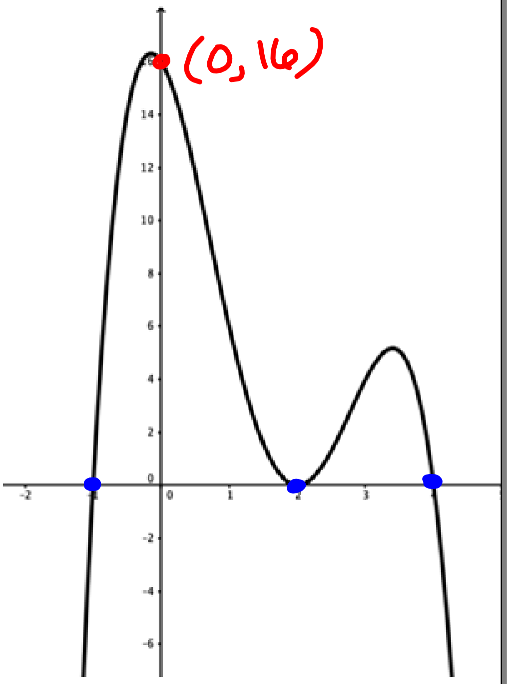
$a=-1$

$$\boxed{C(x)=-1(x+1)(x-2)^2(x-4)}$$

$-1\quad(x)\quad(x)(x)\quad(x)$

$-1x^4$

↙ ↘

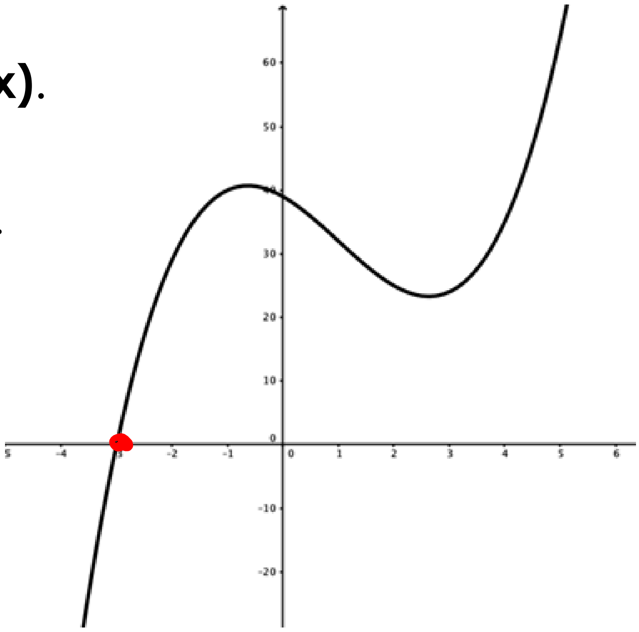


* **Example 4:**

The graph of $k(x)=x^3-3x^2-5x+39$ is shown below. **k(x)** is a 3rd degree polynomial (a "cubic" function). The scale used on the x-axis is 1 unit. The scale used on the y-axis is 10 units.

Determine all the zeros of **k(x)**.

3 zeros



$$
\begin{array}{r}
x^2-6x+13 \\
x+3\,\overline{\big)\,x^3-3x^2-5x+39} \\
+(\cancel{-x^3}+3x^2) \\
\hline
\cancel{-6x^2}-5x+39 \\
+(\cancel{+6x^2}+18x) \\
\hline
\cancel{13x}+\cancel{39} \\
+(\cancel{-13x}-\cancel{39}) \\
\hline
0
\end{array}
$$

$K(x)=(x+3)(x^2-6x+13)$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=1$
$b=-6$
$c=13$

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$

$x=\frac{6\pm\sqrt{36-52}}{2}$

$x=\frac{6\pm\sqrt{-16}}{2}$

$x=\frac{6\pm 4i}{2}$

$x=\frac{6}{2}\pm\frac{4}{2}i$

$x=3\pm 2i$

$$\boxed{\text{zeros: } -3,\ 3+2i,\ 3-2i}$$

*

Example 5:

7 zeros

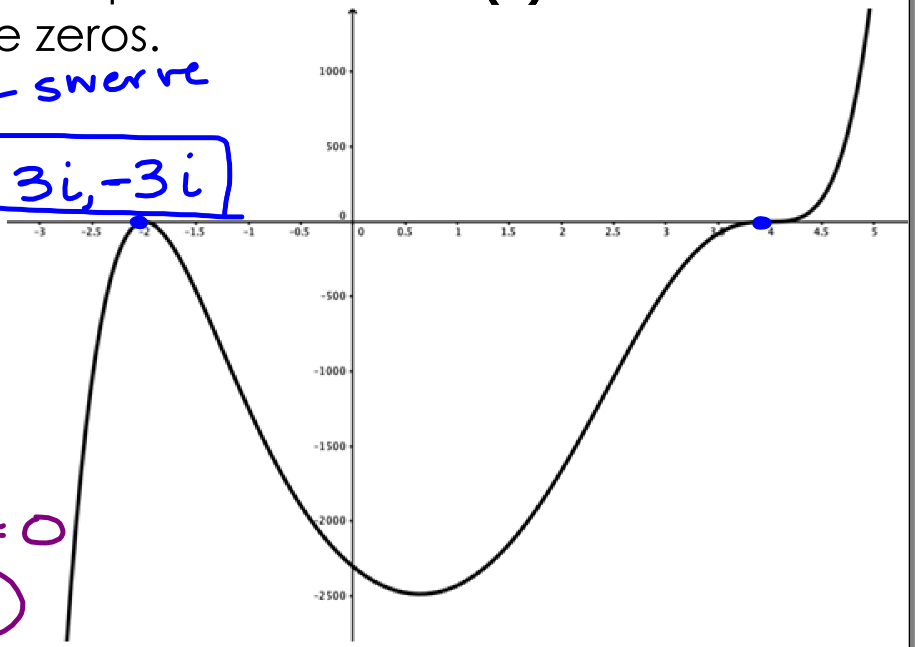
The graph of **L(x)** is shown below. **L(x)** is a 7th degree polynomial.

- **L(x)** has two complex zeros. One of the complex zeros is $x = 3i$. & $-3i$
- The scale used on the x-axis is ½ unit. The scale used on the y-axis is 500 units.

Write the symbolic representation of **L(x)** and determine all the zeros.

bounce swerve

$\boxed{\text{zeros: } -2, 4, 3i, -3i}$



$(x+2)^2(x-4)^3$

$x=3i \quad x=-3i$

$x-3i=0 \quad x+3i=0$

$(x-3i)(x+3i)$

$x^2 \cancel{+3ix-3ix} -9i^2$

$-9 \cdot -1$

(x^2+9)

$(x+2)^2(x-4)^3(x^2+9)$

$(0+2)^2(0-4)^3(0^2+9)$

$(2)^2(-4)^3(9)$

$(4)(-64)(9)$

-2304 ✓

$$\boxed{L(x)=(x+2)^2(x-4)^3(x^2+9)}$$

Practice:

For each of the following functions:

- First, use the symbolic representation of **f(x)** to determine the degree, the total number of zeros the function will have, the y-intercept, and the end-behavior directions;
- Second, use the graph of **f(x)** to determine the number of Real zeros and the number of Complex zeros;
- Third, use the given zero and polynomial long division to determine all remaining zeros of **f(x)**;
- Finally, express **f(x)** in factored form.

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6. $f(x) = x^3 + 2x^2 - 11x - 12$ has one zero at x = 4

Using the symbolic representation of f(x):

A. Degree of f(x): _____ **B.** f(x) will have a total of _____ zeros.

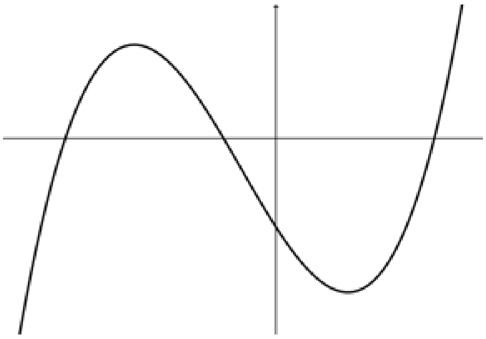
C. y-int of f(x): ________ **D.** End-behavior:

Using the graph of f(x) – note: the scale used on each axis is not provided

E. # of Real zeros: _____ **F.** # of Complex zeros: _____

G. All zeros of f(x): ________________________________

H. f(x) expressed in factored form: ______________________________



7. $f(x) = x^3 - 5x^2 + 17x - 13$ has one zero at x = 1

Using the symbolic representation of f(x):

A. Degree of f(x): _____ **B.** f(x) will have a total of _____ zeros.

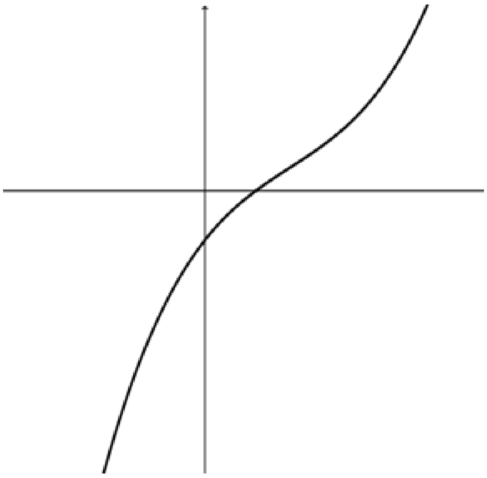
C. y-int of f(x): ________ **D.** End-behavior:

Using the graph of f(x) – note: the scale used on each axis is not provided

E. # of Real zeros: _____ **F.** # of Complex zeros: _____

G. All zeros of f(x): ________________________________

H. f(x) expressed in factored form: ____________________________



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8. $f(x)=x^4+x^3-2x^2+4x-24$ has one zero at x = 2 and one zero at x = -3

Using the symbolic representation of f(x):

A. Degree of f(x): _____ **B.** f(x) will have a total of _____ zeros.

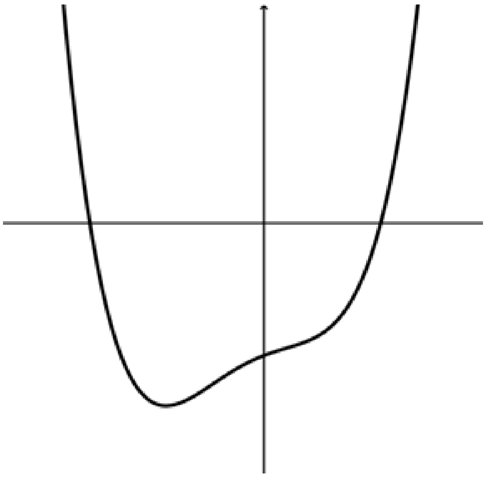
C. y-int of f(x): ________ **D.** End-behavior:

Using the graph of f(x) – note: the scale used on each axis is not provided

E. # of Real zeros: _____ **F.** # of Complex zeros: _____

G. All zeros of f(x): ________________________________

H. f(x) expressed in factored form: _____________________________



*9. $f(x)=x^4-2x^3+10x^2-18x+9$

Using the symbolic representation of f(x):

A. Degree of f(x): 4 **B.** f(x) will have a total of 4 zeros.

C. y-int of f(x): (0,9) **D.** End-behavior: ↖ ↗

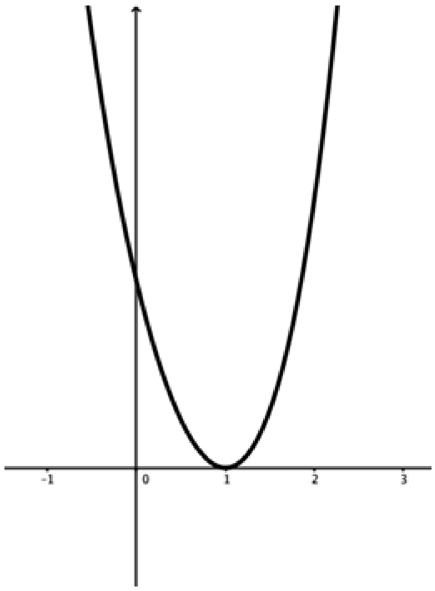
Using the graph of f(x) – note: the scale used on each axis is not provided

E. # of Real zeros: 2 **F.** # of Complex zeros: 2

G. All zeros of f(x): 1, 3i, -3i

H. f(x) expressed in factored form: $f(x)=(x-1)^2(x^2+9)$

$$
\begin{array}{r l}
 & x^3-1x^2+9x-9 \\
x-1 \,\big) & \cancel{x^4}-2x^3+10x^2-18x+9 \\
 & +(\cancel{-x^4}+x^3) \\
 & -1\cancel{x^3}+10x^2-18x+9 \\
 & +(+1\cancel{x^3}-1x^2) \\
 & 9\cancel{x^2}-18x+9 \\
 & +(-9\cancel{x^2}+9x) \\
 & -9\cancel{x}+\cancel{9} \\
 & +(+9\cancel{x}-\cancel{9}) \\
 & 0
\end{array}
$$



$$
\begin{array}{r l}
 & x^2+9 \\
x-1 \,\big) & \cancel{x^3}-\cancel{x^2}+9x-9 \\
 & +(-\cancel{x^3}+\cancel{x^2}) \\
 & 9\cancel{x}-\cancel{9} \\
 & +(-9\cancel{x}+\cancel{9}) \\
 & 0
\end{array}
$$

$$f(x)=x^4-2x^3+10x^2-18x+9$$
$$f(x)=(x-1)(x^3-x^2+9x-9)$$
$$f(x)=(x-1)(x-1)(x^2+9)$$

$x-1=0$ $x-1=0$ $x^2+9=0$

$x=1$ $x=1$ $\sqrt{x^2}=\sqrt{-9}$

$x=\pm 3i$

10. $f(x) = x^3 - 3x^2 - 3x + 1$ (HW)

Using the symbolic representation of f(x):

A. Degree of f(x): _____ **B.** f(x) will have a total of _____ zeros.

C. y-int of f(x): ________ **D.** End-behavior:

Using the graph of f(x) – note: the scale used on each axis is not provided

E. # of Real zeros: _____ **F.** # of Complex zeros: _____

G. All zeros of f(x): ______________________________

H. f(x) expressed in factored form: ____________________________