Exponential Functions 7 - Solving Exponential Equations

Standards: **F-LE.4**, **F-BF.5**

Learning Target

-How do you solve exponential equations?

Solve: $3^x = 5$

We know that $3^1 = 3$ and $3^2 = 9$ so our value for x is somewhere between 1 & 2. But since x is in the exponent (exponential equation) how do we get x by itself? How do we "get it down" from there?

We need to do the inverse or "opposite" of an exponential function to "undo" it. We need to **take the log** of both sides. (erase to show)

There are two different ways you can do this.

Method1:

(erase to show)

$$3^x = 5$$

$$\log 3^x = \log 5$$ log both sides

$$x \cdot \log 3 = \log 5$$ use the Power Property

$$\frac{x \cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 5}{\log 3}$$ Divide both sides by log3

$$x = \frac{\log 5}{\log 3} \approx 1.465$$ simplify

using calculator

Method2:

(erase to show)

$$3^x = 5$$

$$\cancel{\log_3 3^x} = \log_3 5 \qquad \text{log base 3 both sides}$$

$$x = \log_3 5 \qquad \text{Inverse property of logs}$$

$$x = \frac{\log 5}{\log 3} \approx 1.465 \qquad \text{Use change-of-base formula}$$

(Remember, exponential & logarithmic functions are inverses and "undo" each other.)

$$\cancel{\log_b b^x} = x$$

<u>Example1</u>: Solve. Round your answer to 3 decimal places.
Check your answer.

a) $4^x = 15$

#1

$\log 4^{x} = \log 15$

$\dfrac{x \cdot \log \cancel{4}}{\cancel{\log 4}} = \dfrac{\log 15}{\log 4}$

$\boxed{x \approx 1.953}$

b) $2^x = 7$

#2

$\cancel{\log_2 2}^{x} = \log_2 7$

$x = \dfrac{\log 7}{\log 2}$

$\boxed{x \approx 2.807}$

We can get a little more complex.
Just remember to work backwards (SADMEP)

Example2: Solve. Round to 3 decimal places.
Check your answer.

a) $5^{x+2} + \cancel{3} = 25$
$\quad\ \ -\cancel{3} \quad -3$

#1

$5^{x+2} = 22$

$\log 5^{x+2} = \log 22$

$\dfrac{(x+2)\cdot \cancel{\log 5}}{\cancel{\log 5}} = \dfrac{\log 22}{\log 5}$

$x+\cancel{2} = \dfrac{\log 22}{\log 5} - 2$
$\ \ -\cancel{2}$

[$\dfrac{\log 22}{\log 5}$ enter -2 enter]

$\boxed{x \approx -0.079}$

b) $\cancel{8} + 10^{5x+4} = 35$
$-\cancel{8} \qquad\qquad -8$

#2

$10^{5x+4} = 27$

$\cancel{\log_{10} 10}^{5x+4} = \log_{10} 27$

$5x+\cancel{4} = \dfrac{\log 27}{\log 10} - 4$
$\quad -\cancel{4}$

$\dfrac{\cancel{5}x}{\cancel{5}} = \dfrac{\dfrac{\log 27}{\log 10} - 4}{5}$

[$\dfrac{\log 27}{\log 10}$ enter -4 enter $\div 5$ enter]

$\boxed{x \approx -0.514}$

c) $\frac{\cancel{2}e^x}{\cancel{2}} = \frac{10}{2}$

#1

$e^x = 5$

$\log e^x = \log 5$

$\frac{x \cdot \cancel{\log e}}{\cancel{\log e}} = \frac{\log 5}{\log e}$

$\boxed{x \approx 1.609}$

d) $\frac{\cancel{5}e^{x+1}}{\cancel{5}} = \frac{20}{5}$

#2

$e^{x+1} = 4$

$\cancel{\log_e} e^{x+1} = \log_e 4$

$x + \cancel{1} = \ln 4 - 1$

$\cancel{-1}$

[ln 4 enter -1 enter]

$\boxed{x \approx 0.386}$

<u>Example3</u>: Solve. Round to 3 decimal places.
Check your answer.

a) $2^{4x} = 32^{x-1}$

#1

$\log 2^{4x} = \log 32^{x-1}$

$\dfrac{(4x)\cdot\cancel{\log 2}}{\cancel{\log 2}} = \dfrac{(x-1)\cdot\log 32}{\log 2}$

$4x = (x-1)\cdot 5$

$4x = 5x - 5$

$-5x \quad -\cancel{5x}$

$\dfrac{\cancel{-}x}{\cancel{-}1} = \dfrac{\cancel{-}5}{\cancel{-}1}$

$\boxed{x = 5}$

b) $9^{2x} = 81^{3x-2}$

#2

$\cancel{\log_9 9}^{2x} = \log_9 81^{3x-2}$

$2x = (3x-2)\cdot\log_9 81$

$2x = (3x-2)\cdot\dfrac{\log 81}{\log 9}$

$2x = (3x-2)\cdot 2$

$2x = 6x - 4$

$-6x \quad -\cancel{6x}$

$\dfrac{-\cancel{4}x}{-\cancel{4}} = \dfrac{-4}{-4}$

$\boxed{x = 1}$

Now that we know how to solve for an exponential equation, we can find the exact solutions for those real-world problems we worked on a few weeks ago!

Example 4:

The amount, in grams, of a certain radioactive element that is left after t-years is modeled by

$$P(t) = 500(.8)^t$$

Approximately when would you have only 10 grams left?

find t.

$P(t) = 10$

$$\frac{10}{500} = \frac{\cancel{500}(0.8)^t}{\cancel{500}}$$

$$0.02 = 0.8^t$$

#1

$$\log 0.02 = \log 0.8^t$$

$$\frac{\log 0.02}{\log 0.8} = \frac{t \cdot \cancel{\log 0.8}}{\cancel{\log 0.8}}$$

$$t \approx 17.531$$

#2

$$\log_{0.8} 0.02 = \cancel{\log_{0.8}} 0.8^t$$

$$t = \frac{\log 0.02}{\log 0.8}$$

$$t \approx 17.531$$

It would take about 17.5 years.

Example 5:

You bought a new car for $24,900. The value of the car depreciates 11% every year for *t* years after it was purchased. After how many years will the car be worth half its purchase price? find t

$\frac{24{,}900}{2} = 12{,}450$

$$\boxed{\begin{aligned} y &= a(1-r)^t \\ y &= 12{,}450 \\ a &= 24{,}900 \\ r &= 11\% = 0.11 \\ t &= t \end{aligned}}$$

$$12{,}450 = 24{,}900(1-0.11)^t$$

$$\frac{12{,}450}{24{,}900} = \frac{\cancel{24{,}900}(0.89)^t}{\cancel{24{,}900}}$$

$$0.5 = 0.89^t$$

#1

$$\log 0.5 = \log 0.89^t$$

$$\frac{\log 0.5}{\log 0.89} = \frac{t \cdot \cancel{\log 0.89}}{\cancel{\log 0.89}}$$

$$t \approx 5.948$$

#2

$$\log_{0.89} 0.5 = \cancel{\log_{0.89} 0.89^t}$$

$$t = \frac{\log 0.5}{\log 0.89}$$

$$t \approx 5.948$$

$$\boxed{\text{About 5.9 years / 6 years}}$$