Quadratics 2b - Quadratic Function: Graphing Factored Form

Standards: **F-IF.4** & **F-IF.7**

GLOs: #3 Complex Thinker

Math Practice: Look for and make use of structure

HW: WS #9 (graph on graph paper!)

Learning Target:

How does Factored form help in graphing a quadratic?

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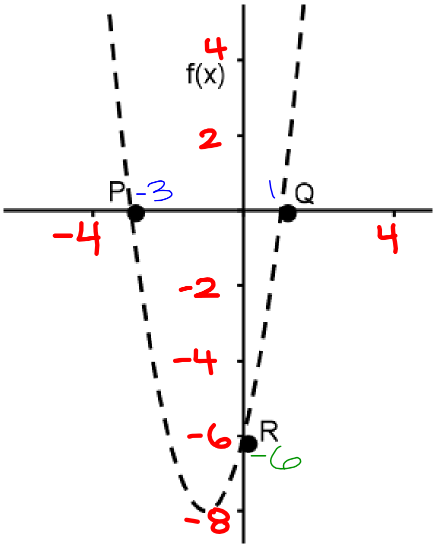
Below is the graph for a quadratic function, with the x and y intercepts clearly marked.



1. Since the units are not marked, it is impossible to identify both coordinates of each point, but it is always possible to identify one of the coordinates for an intercept. Fill in one coordinate for each of the points below.

a) P = (P, 0) **b)** Q = (Q, 0)

c) R = (0, R)

(erase to show)

So based on this we know:

The __x__ coordinate of a y-intercept is always __0__.
The __y__ coordinate of an x-intercept is always __0__.

> To find the **x-intercept(s)**: Plug in __0__ for __y__ and solve.

> To find the **y-intercept**: Plug in __0__ for __x__ and solve.

2. The formula for the function f on the previous page is

$$y \rightarrow f(x) = 2(x-1)(x+3)$$

Use this formula to find the missing coordinates of the x – intercepts and the y – intercept. Label the tick marks on each axis with an appropriate scale (i.e. each tick mark represents what quantity for each axis?).

y-int: $f(0)=2(0-1)(0+3)$

$f(0)=2(-1)(3)$

$f(0)=-6$

$\boxed{\text{y-int}: (0,-6)}$

x-int: $0=2(x-1)(x+3)$

$\cancel{2=0}$ $\quad x-1=0$, $+1 \; +1$, $x=1$ $\quad x+3=0$, $-3 \; -3$, $x=-3$

$\boxed{\text{x-int}: (-3,0) \text{ \& } (1,0)}$

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Definition: The x-coordinate of an x-intercept is referred to as a **zero** for *f*.

solutions → zeros → x-intercepts
x = −3   −3   (−3, 0)

Careful, there is an obvious connection between x – intercepts and zeros, but the x – intercept is a point and the zero is a number, so they are not the same.

In order to find the zeros for a quadratic function defined in the form $f(x)=a(x-s)(x-t)$

set **a(x-s)(x-t)** = 0 and solve for x. This requires using the Zero-Product Property, and solving each for x.

3. Given $g(x)=-(x-1)(x-5)$

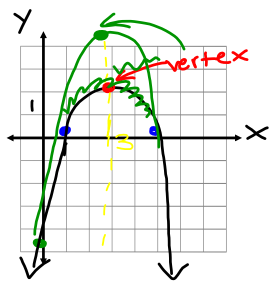
Use this formula to find the x – intercepts & the y – intercept. State the zeros of g(x). Show all work!

y-int: $g(0)=-(0-1)(0-5)$
$g(0)=-(-1)(-5)$
$g(0)=-5$
$\boxed{\text{y-int: } (0,-5)}$

x-int: $0=-(x-1)(x-5)$
$\cancel{-1}\neq 0$ $x-1=0$, $x=1$ $x-5=0$, $x=5$
$\boxed{\text{x-int: } (1,0) \text{ \& } (5,0)}$
$\boxed{\text{zeros: } 1 \text{ \& } 5}$

Reflection:
Now that you know how to find three points,
where do you think the vertex is? Sketch it to see.
What is the relationship between the zeros for g(x) and the location of its vertex?
How could we possibly find the exact values?

The vertex is in the middle of the zeros/x-intercepts.



4. Use your answers in Problem3 to find the x-coordinate of the vertex of g(x). Use your x-coordinate to then find the y-coordinate.

x-coordinate of vertex: 3

y-coordinate:

$g(3)=-(3-1)(3-5)$
$g(3)=-(2)(-2)$
$g(3)=4$

$\boxed{\text{vertex: } (3,4)}$

(erase to show)

Summary – What we should know thus far about the graph of f(x) = a(x – s)(x – t):

1. Since f is a quadratic, its graph is a **parabola**.
2. In order to find its x – intercepts, set *y*, or *f(x)*, equal to 0, then use the Zero-Product Property to find the zeros of the equation.
   *REMEMBER: Zeros are expressed as a number, while intercepts are written as an ordered pair.
3. In order to find the y – intercept, simply evaluate the function at **x = 0**.
4. If a > 0 then the parabola will be concave **up**.
5. If a < 0 then the parabola will be concave **down**.
6. The x – coordinate of the vertex is located **halfway** between the **x-intercepts.**
7. The y – coordinate of the vertex can be found by evaluating f at the x – coordinate.

Note: Another way of thinking about the relationship between factors of a quadratic function and its zeros is the following: **factors yield zeros** (simply set each factor to zero and solve x).

Sep 8-10:51 PM

Example 1: Graph $f(x) = 2(x-1)(x+5)$

Be sure to indicate both coordinates of each intercept and the vertex.

Solution: f is a quadratic, so we are looking to graph a parabola.

a. Find the **zeros:** Zeros occur at points where f(x) = 0 (x-intercepts on the graph). Set 2(x – 1)(x+5) = 0. By the Zero-Product Property this implies either x - 1 = 0 or x + 5 = 0, which implies x = 1 or x = -5.

b. Using these zeros, locate the x-intercepts on the graph at the points (1, 0) and (-5, 0).

c. Using the fact that a = 2, we know that the parabola opens upward.

d. Find thex-coordinate of the **vertex**, which is located halfway between the two zeros. The x-coordinate of the vertex is therefore -2 (the average of 1 and -5 is -2).

e. The vertex is therefore at the point (-2, f(-2)).

f(-2) = 2(-2 – 1)(-2 + 5) = 2(-3)(3) = -18.

Locate the vertex on the graph at the point (-2, -18).

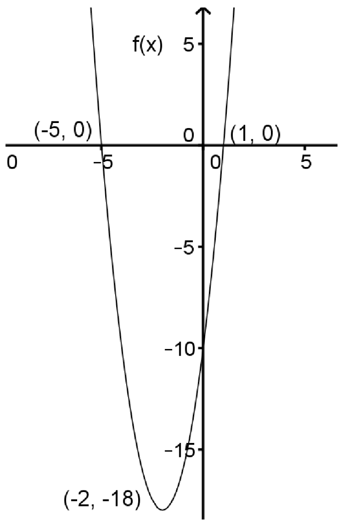
f. Find the **y-intercept**: f(0) = 2(0 - 1)(0 + 5)=2(-1)(5) = -10. Locate the y-intercept on the graph at the point (0, -10).

g. Graph f. Find additional points if necessary.

h. Notice, this is the same as graphing

$f(x) = 2(x^2 + 4x - 5) = 2x^2 + 8x - 10$,

but you can see that it is much easier graphing f in its factored form.



Example 2:

Graph: $f(x) = -(x-2)(x+4)$

parabola concave down

y-int: $f(0) = -(0-2)(0+4)$

$f(0) = -(-2)(4)$

$f(0) = 8$

$\boxed{\text{y-int: } (0,8)}$

x-int: $0 = -(x-2)(x+4)$

$\cancel{-1=0}$ $x - \cancel{2} = 0$ $x + \cancel{4} = 0$

$+\cancel{2} \ +2$ $-\cancel{4} \ -4$

$x = 2$ $x = -4$

$\boxed{\text{x-int: } (2,0) \,\&\, (-4,0)}$

vertex:

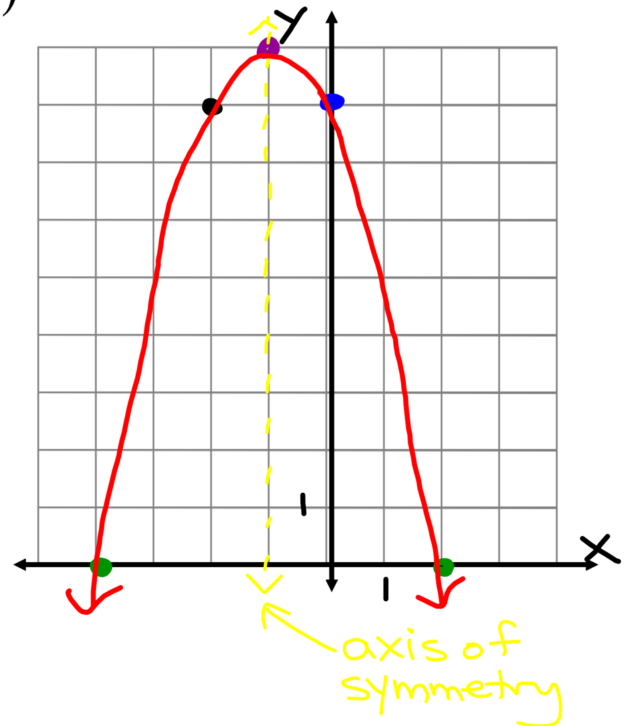
x: $\frac{2+-4}{2} = \frac{-2}{2} = -1$

y: $f(-1) = -(-1-2)(-1+4)$

$f(-1) = -(-3)(3)$

$f(-1) = 9$

$\boxed{\text{Vertex: } (-1,9)}$



Domain: all real numbers

Range: $y \leq 9$

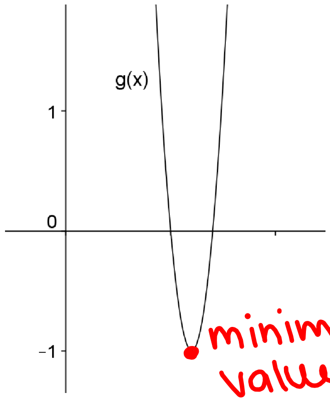
$f(x) \leq 9$

Challenge / Bell Ringer:

Suppose $f(x)=2(x-2)(x-4)$

and the graph of the quadratic function *g* is given below. Which has the smaller minimum value, *f* or *g*? Explain.

vertex



minimum value, -1

Find vertex of f(x):

x-int: $0=2(x-2)(x-4)$

$\cancel{2=0}$ $\quad x-2=0 \quad x-4=0$

$\quad +2 \;\; +2 \quad\quad +4 \;\; +4$

$x=2 \quad\quad x=4$

zeros @ 2, 4

Vertex:

x: $\dfrac{2+4}{2}=\dfrac{6}{2}=3$

y: $f(3)=2(3-2)(3-4)$

$f(3)=2(1)(-1)$

$f(3)=-2$

$\boxed{\text{vertex: } (3,-2)}$ ← minimum value at -2

Since f(x) has a minimum value, -2 which is lower than g(x) minimum value, -1, f(x) has the smaller minimum value.