Radicals 3 - nth Roots & Rational Exponents

Standards:

N-RN.1

N-RN.2

Learning Targets

What does a fraction exponent mean?

How do you evaluate nth roots?

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(erase to show)

n^{th} root of $a \longrightarrow \sqrt[n]{a}$

n is the **index** of the radical

Example 1: Evaluate without a calculator

a) $\sqrt{81} = \sqrt[2]{81} = 9$

$?^2 = 81$ $9^2 = 81$ ✓

b) $\sqrt[3]{27} = 3$

$?^3 = 27$ $3^3 = 27$ ✓

c) $\sqrt[4]{625} = 5$

$?^4 = 625$ $5^4 = 625$ ✓

d) $\sqrt[5]{32} = 2$

$?^5 = 32$ $2^5 = 32$ ✓

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Now try these problems with a calculator.

Scientific calculator: use the button $\sqrt[x]{y}$

TI-83/84: On the home screen type in index #
Press Math button
Pick #5: $\sqrt[x]{}$
Fill in radicand

a) $\sqrt{81} = \sqrt[2]{81} = 9$

2 MATH 5 (81)

b) $\sqrt[3]{27} = 3$

3 MATH 5 (27)

c) $\sqrt[4]{625} = 5$

4 MATH 5 (625)

d) $\sqrt[5]{32} = 2$

5 MATH 5 (32)

Sometimes it's more convenient to write a root as an exponent...but how?

$$\sqrt{a}\cdot\sqrt{a}=a$$

$$a^{k}\cdot a^{k}=a$$

$$a^{2k}=a^{1}$$

$$2k=1$$

$$k=\frac{1}{2}$$ Therefore, $\sqrt[2]{a}=a^{1/2}$

$$\sqrt[3]{x}=x^{1/3}$$

$$\sqrt[4]{10}=10^{1/4}$$

$$\sqrt[n]{a}=a^{1/n}$$

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We can also have different exponents than 1.

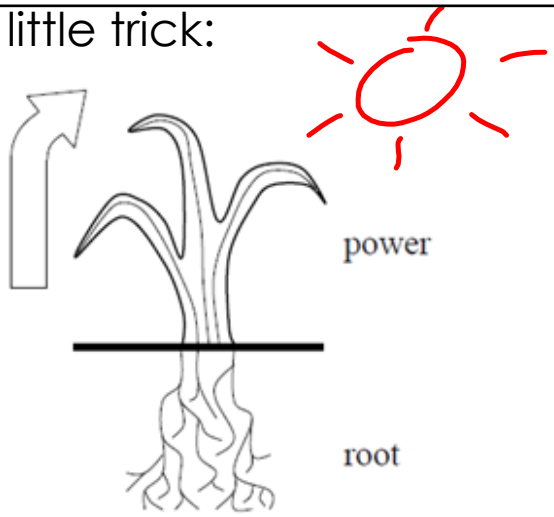
$$\left(\sqrt[n]{a}\right)^m = \left(a^{1/n}\right)^m = a^{m/n}$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\left(a^{1/n}\right)^m} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$$

rational exponent

radical notation

A little trick:



$$x^{\frac{\text{power}}{\text{root}}} = \sqrt[\text{root}]{x^{\text{power}}} \quad \text{or} \quad \left(\sqrt[\text{root}]{x}\right)^{\text{power}}$$

$$x^{\frac{1}{3}} \rightarrow \sqrt[3]{x} \quad \text{or} \quad x^{\frac{\text{power}}{\text{root}}} \rightarrow \sqrt[3]{x^1}$$

rational exponent

radical notation

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Example 2: Simplify.

a) $64^{1/3}$ power: 1, root: 3

$\sqrt[3]{64}$

$?^3 = 64$

4^1

4

b) $(-125)^{1/3}$ power: 1, root: 3

$\sqrt[3]{-125}$

$?^3 = -125$

$(-5)^1$

-5

c) $16^{3/4}$ power: 3, root: 4

$\left(\sqrt[4]{16}\right)^3$

$?^4 = 16$

$(2)^3$

8

d) $27^{-4/3}$

$\frac{1}{27^{4/3}}$ power: 4, root: 3

$\frac{1}{\left(\sqrt[3]{27}\right)^4}$

$\frac{1}{(3)^4}$

$\frac{1}{81}$

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Entrance Pass

(Due the beginning of next class)

Rewrite using rational exponents.

1 $\left(\sqrt[3]{11}\right)^1$ $11^{1/3}$

$\left(\sqrt[n]{a}\right)^m = a^{m/n}$

Rewrite using radical notation.

2. $10^{3/7}$ $\left(\sqrt[7]{10}\right)^3$

Evaluate the expression without using a calculator.

3. $9^{3/2}$ $\left(\sqrt[2]{9}\right)^3$

$(3)^3$

$\boxed{27}$

If you have a calculator, you may have to rewrite the n^{th} root.
(depending on your calculator)

Ex: Try $\left(\sqrt[3]{-7}\right)^2$ or $(-7)^{2/3}$

$\left(\sqrt[5]{-2}\right)^7$ $(-2)^{7/5}$

-2.6390

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Solving Eqns using nth roots

To solve an eqn with x^n, take the nth root of both sides.

Real nth Roots

- If *n* is **odd**, then *a* has **1** real nth root (solution) $\sqrt[n]{a} = a^{1/n}$
- If *n* is **even** and *a*=0, then *a* has **1** nth root $\sqrt[n]{0} = 0^{1/n} = 0$
- If *n* is **even** and *a*>0 (positive), then *a* has **2** real nth roots $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If *n* is **even** and *a*<0 (negative), then *a* has **no** real roots.

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Example 3: Solve & round to 2 decimal places.

a) $\frac{2x^3}{2} = \frac{92}{2}$

$\sqrt[3]{x^3} = \sqrt[3]{46}$

$46^{1/3}$

$x = 3.58$

b) $\sqrt[3]{(x+4)^3} = \sqrt[3]{-27}$

$(-27)^{1/3}$

$x + 4 = -3$

$-4 \quad -4$

$x = -7$

c) $\sqrt[4]{(x-1)^4} = \sqrt[4]{12}$

$12^{1/4}$

$x - 1 = \pm 1.8612$

$+1 \quad +1$

$x = 1 \pm 1.8612$

$x = 1 + 1.8612 \quad x = 1 - 1.8612$

$x = 2.86$ & 0.86

d) $\sqrt[6]{(x-1)^6} = \sqrt[6]{28}$

$28^{1/6}$

$x - 1 = \pm 1.7425$

$+1 \quad +1$

$x = 1 \pm 1.7425$

$x = 1 + 1.7425 \quad x = 1 - 1.7425$

$x = 2.74$ & 0.74

Exit Pass

Determine whether each expression is equivalent to $\sqrt[9]{4^3}$ → $(\sqrt[9]{4})^3$ → $4^{3/9}$ → $4^{1/3}$

Select Yes or No for each expression.

		Yes	No
A.	$4^{\frac{1}{3}}$	✓	
B.	36^3		✓
C.	4^{27}		✓
D.	$4^{\frac{3}{9}}$	✓	
E.	$\sqrt[3]{4}$	✓	