Polynomial 6a - Polynomial Long Division

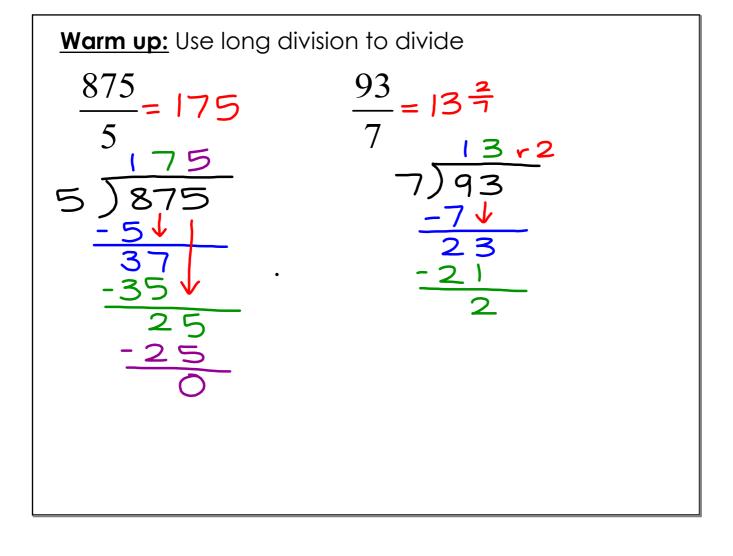
Standards: A-APR.2, A-APR.6, A-REI.4

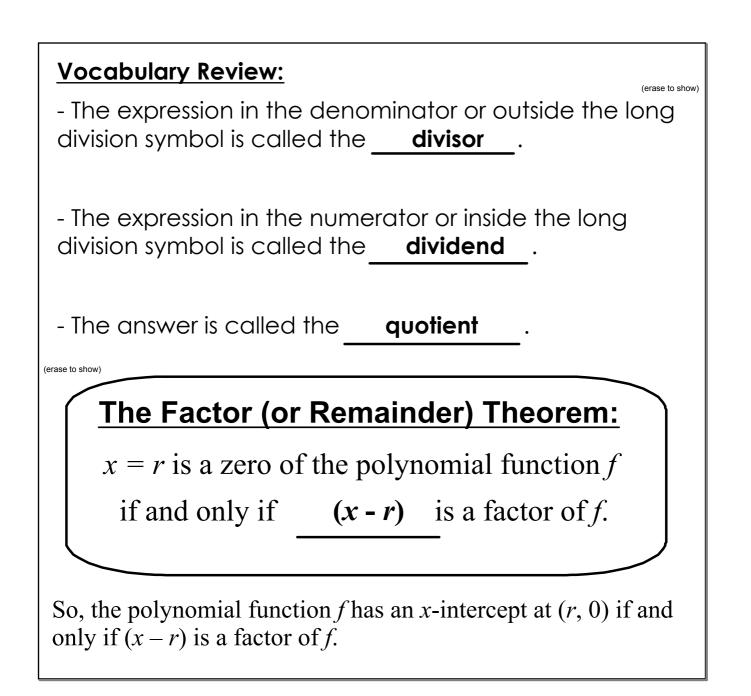
GLO: #3 Complex Thinker

Math Practice: #7 Look for and make use of structure

Learning Target:

How is long division used with polynomials?





Knowing the zeros for a polynomial makes it easy to roughly sketch its graph, and the graph in turn allows us to better interpret the meaning of the symbolic form.

Finding zeros is *critical* for understanding the behavior of a polynomial, particularly its changes in direction.

This was true for quadratic functions as well and in previous lessons we were able to easily locate zeros because we could visually locate them on a given graph or because the symbolic forms were given in factored form.

Now we need a method to find the zeros of dup polynomial function given in standard form IF we know at least one zero for the function.

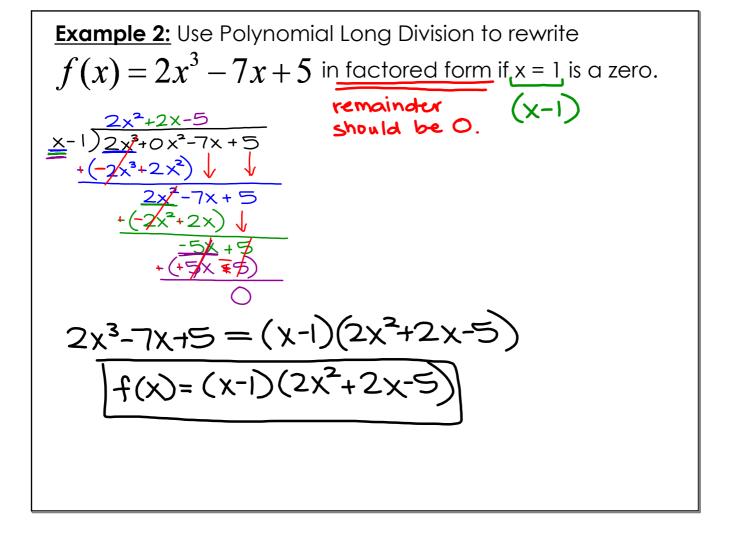
This method is called **Polynomial Long Division**

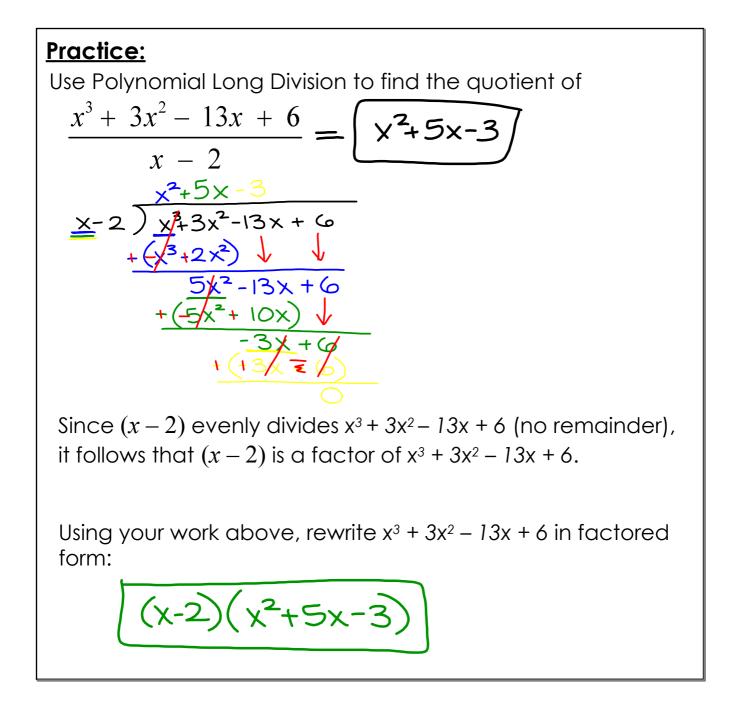
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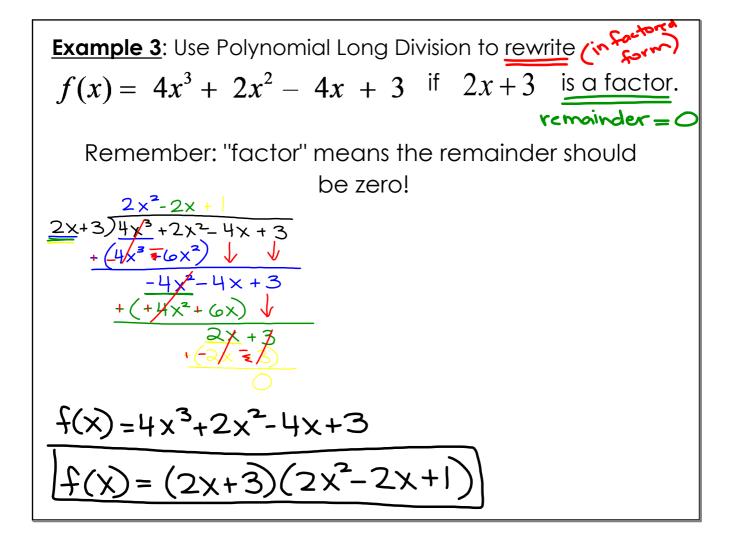
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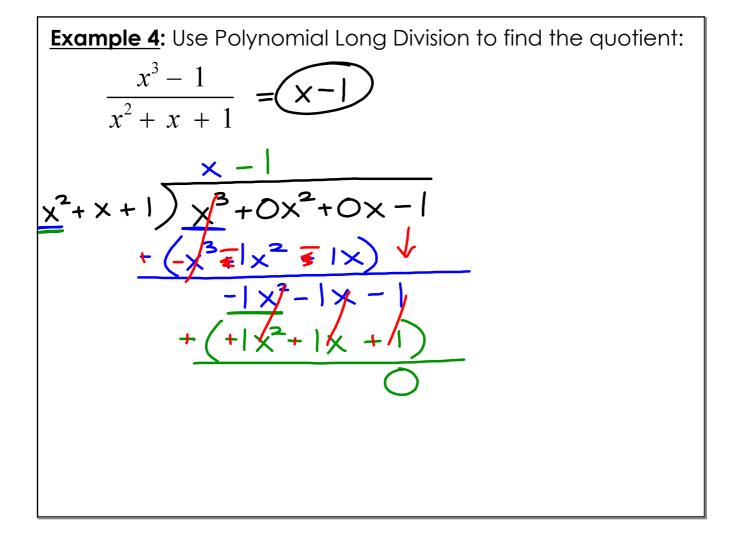
Example 1: Rewrite $f(x) = x^3 - 4x^2 + 2x + 3$ in factored form if we know x = 3 is one of the zeros. Notice: x = 3 is a zero means (x - 3) is a factor. So how do we determine the other factor(s)? $(x-3)(_?_) = x^3 - 4x^2 + 2x + 3$ We can use long division! $\frac{x^3 - 4x^2 + 2x + 3}{x - 3}$ Original Problem $x-3)\overline{x^3-4x^2+2x+3}$ Rewrite in long division form For this problem we first want to determine what we need to multiply (x - 3) by to get x^3 . Of course the answer is x^2 , but when we multiply x - 3 by x^2 we get more than x³. Subsequent steps are given below. *Place* x^2 *in the quotient* $\underline{x}^{-3} \underbrace{x^{3} - 4x^{2} + 2x + 3}_{+ \underbrace{x^{3} + 3x^{2}}_{+} \underbrace{x^{3} + 3x^{2}}_{+} \underbrace{x^{3} + 3x^{2}}_{-} \underbrace{x^{3} - 4x^{2} + 2x + 3}_{-} Multiply x^{2} by x - 3 and subtract$ $x-3\sqrt{x^3-4x^2+2x+3}$ We now need $-x^2$, so we place -x in the quotient $-(x^3-3x^2)$ Multiply -x by x - 3 and subtract Continuing, we finally place -1 in the quotient to get $x^2 - x - 1$ $x-3) \overline{x^3-4x^2+2x+3}$ $-(x^3-3x^2)$ $-x^{2}+2x+3$ $-(-x^2+3x)$ We now need -x, so we place -1 in the quotient -(-x+3) Multiply -1 by x-3 and subtract Since the remainder is zero we know that (x - 3) evenly divides $x^3 - 4x^2 + 2x + 3$, which implies that $(x^2 - x - 1)$ is a factor of $x^3 - 4x^2 + 2x + 3$. In other words, $x^{3}-4x^{2}+2x+3=(x-3)(x^{2}-x-1)$ Later, when we need to graph, we can use the methods we already know (Factoring or Quadratic Formula) to find the other zeros of $x^2 - x - \tilde{1}$.

<u>Caution</u>: Notice the use of parentheses throughout the subtraction steps. When performing long division with numbers there is usually no need for parentheses, but since polynomials have multiple terms that need to be subtracted from another polynomial, parentheses are essential. Below is an example of what to do when the dividend does not contain all powers of x.









Example 5: Use Polynomial Long Division:

$$\begin{pmatrix}
5x^{3} + 12x^{2} + 7x - 1 \\
5x^{2} + 2x + 3 \\
x + 2 \\
5x^{2} + 2x + 3 \\
x + 2 \\
5x^{2} + 12x^{2} + 7x - 1 \\
+ (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
+ (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
+ (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
+ (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
+ (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
x + (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
x + (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
x + (-5x^{3} + 10x^{2}) \\
x + (-5x^{3} + 10x^{2}) \\
x + 2x^{2} + 7x - 1 \\
x + (-5x^{3} + 10x^{2}) \\
x + (-5x^$$

What if your remainder was not zero?

Short answer: then the divisor is NOT a factor of the dividend, and setting the divisor to zero will not give you a zero for the dividend.

Remember, only when the remainder is zero do we know (x - r) to be a factor.

