Exponential Functions 4 - The Number e

Standards: **A-CED.1, A-CED.4, A-SSE.3c, F-IF.8b**

Learning Targets:

- What is the number e?
- How do we use the number e in the real world?

(erase to show)

Like π and i, e is a famous number called the **Natural Base e** or the **Euler Number**.

Just as with π, e is also an irrational number.

As n approaches ∞, $\left(1+\frac{1}{n}\right)^n$ approaches

$e \approx 2.718281828459045235360287471 3...$

At this point it is of little concern how to compute this value, since you will find it on your calculator.

(look for the button $\boxed{e^x}$)

Example 1: Evaluate. Round to three decimal places.

a) e^3

e ^ (3) $\boxed{\text{enter}}$

$\boxed{20.086}$

b) $e^{-0.12}$

$\boxed{0.887}$

Example 2: Simplify in terms of e.

a) $e^3 \cdot e^7$

e^{3+7}

$\boxed{e^{10}}$

b) $\dfrac{24e^{81}}{8e^5}$

$\dfrac{24}{8} \cdot \dfrac{e^{81}}{e^5}$

$\dfrac{3}{1} \cdot e^{81-5}$

$\boxed{3e^{76}}$

c) $\left(2e^{-5x}\right)^{-2}$

$2^{-2} \cdot e^{-5x \cdot -2}$

$\left(\frac{1}{2}\right)^2 \cdot e^{10x}$

$\frac{1}{4} \cdot \frac{e^{10x}}{1}$

$\boxed{\dfrac{e^{10x}}{4}}$

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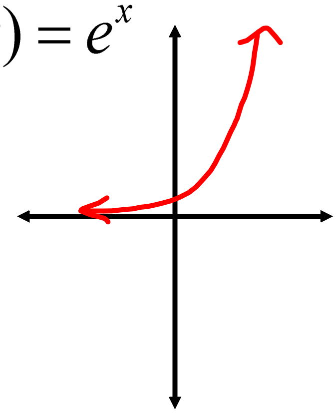
(erase to show)

The function $\boxed{f(x) = ae^{rx}}$ is the natural base exponential form.

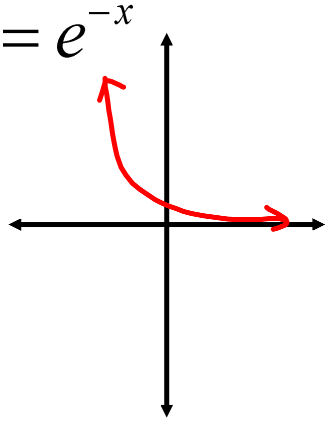
If **r > 0** then it is a **growth** function

$$f(x) = e^{x}$$



If **r < 0** then it is a **decay** function

$$f(x) = e^{-x}$$

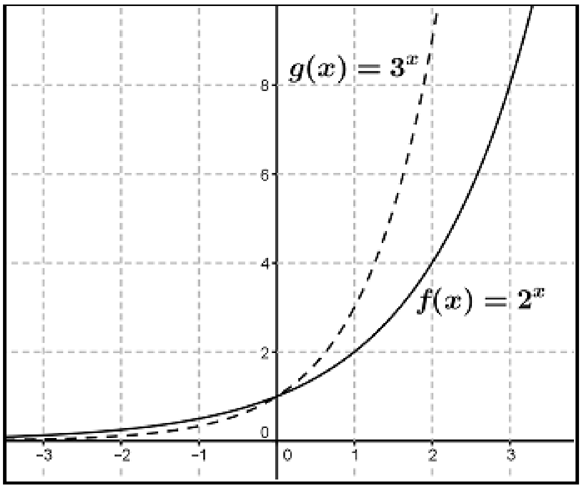


Is the function exponential decay or growth? $f(x) = \frac{2}{3}e^{4x-1}$

exponential growth

Graphing *e*

To the right are the graphs of $f(x) = 2^x$ and $g(x) = 3^x$. Using the fact that $e \approx 2.71828$, on the same axes draw your best guess of the graph of $h(x) = e^x$.



1. For large negative values of x, the value for e^x is close to $y =$ <u>0</u>. (asymptote)

2. Does $h(x) = e^x$ have any x-intercept(s)? If so, where.

No, e^x will never cross the x-axis.

3. Does $h(x) = e^x$ have a y-intercept? If so, where.

Yes, at (0,1).

$$f(x) = ae^{r(x-h)} + k$$

(don't distribute r!)

<u>To graph:</u>

1. Sketch $f(x) = ae^{rx}$
2. Shift it h horizontally
   k vertically

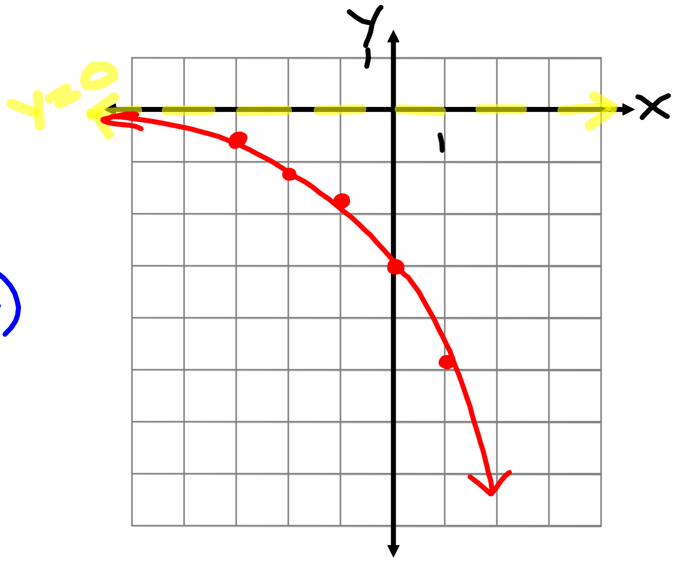
Example 3a: Graph. Then state the domain,range, asymptote & y-int.

$$f(x) = -3e^{0.5x}$$

$f(x)=-3e^{0.5x}$

$h=0$
$k=0$
(no shift)

x	y
-1	-1.82
0	-3
1	-4.95
~~2~~	~~-8.16~~
-2	-1.10
-3	-0.67



domain: $\mathbb{R}$

range: $y<0$

asymptote: $y=0$

y-intercept: $(0,-3)$

Example 3b: Graph. Then state the domain,range, asymptote & y-int.

$$f(x) = e^{0.4(x+1)} - 2$$

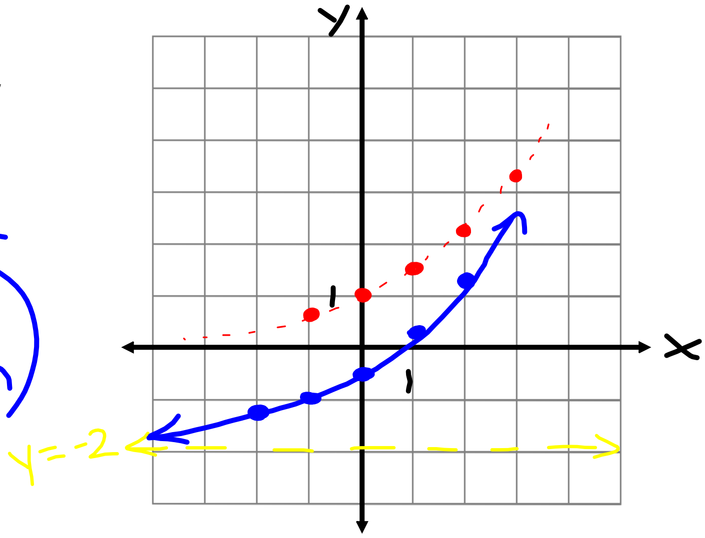
$f(x) = e^{(0.4x)}$

x	y
-1	0.67
0	1
1	1.49
2	2.23
3	3.32

$h = -1$

$k = -2$

(1 left
2 down)



domain: $\mathbb{R}$

range: $y > -2$

asymptote: $y = -2$

y-intercept: $(0, -0.508)$

$= e^{0.4(0+1)} - 2$

$= e^{0.4(1)} - 2$

$= e^{0.4} - 2$

$= -0.508$

Compounded Interest

$$\boxed{A = P\left(1+\frac{r}{n}\right)^{nt}}$$

Recall that the more we compounded per year the more interest you earned.

As $n \rightarrow \infty$ the interest formula approximates the formula for

Continuously Compounded Interest

P = Principle
r = rate (decimal)
t = time (yrs)

$$\boxed{A = Pe^{rt}}$$

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Example 4:
You deposit \$1500 in an account that pays 35% annual interest compounded continuously. What is the balance after 5 years?

$A = Pe^{rt}$

$P = 1500$

$r = 3.5\% = 0.035$

$t = 5$ yrs

$A = 1500 \cdot e^{0.035(5)}$

$= 1500 \cdot e^{0.175}$

$\boxed{\$1786.87}$

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