Radicals 1 - The Square Root Graph

Standard: F-IF.7
Math Practice: Look for and make use of structure
GLOs: #3-Complex Thinker

Learning Target:
- How do you determine the domain & range of a square root function?
- How do you graph a square root function?

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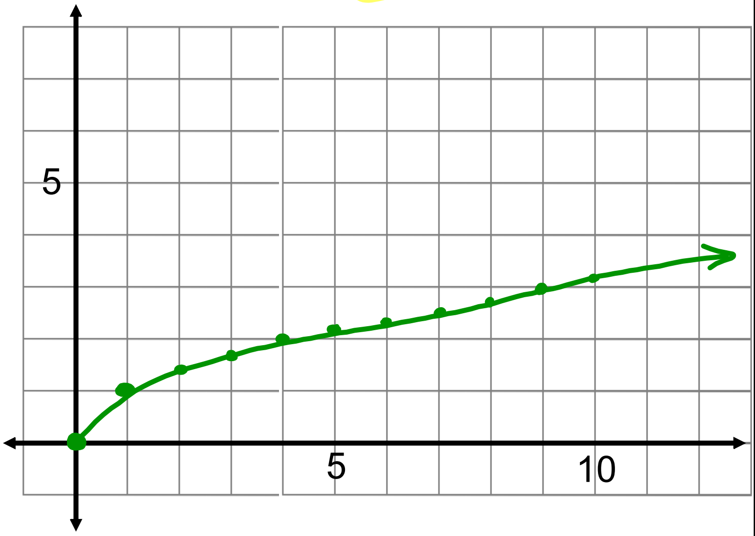
In previous classes and throughout this year, we have studied linear functions, quadratic functions, and polynomial functions. Now, we will begin the study of a new type of a function - radical functions. They are called radical functions because they have a radical symbol, $\sqrt{}$, in them.

The simplest of radical functions is the square root function,

$$f(x)=\sqrt{x}$$

1. Fill in the table below. Leave all decimal values accurate to four decimal places.
Use the values to draw the graph of $f(x)=\sqrt{x}$

x	$f(x)$
0	0
1	1
2	1.4142
3	1.7321
4	2
5	2.2361
6	2.4495
7	2.6458
8	2.8284
9	3
10	3.1623



2. Why should we not use negative values of x with $f(x) = \sqrt{x}$?

Square rooting a negative number outputs an imaginary value, which cannot be graphed.

3. What is the **domain** of $f(x) = \sqrt{x}$?
(The domain of a function is its set of *inputs,* all possible numbers that could be used as an input value.)

- All real numbers greater than or equal to 0.
- $\{x : x \geq 0\}$
- $[0, \infty)$

4. What is the **range** of $f(x) = \sqrt{x}$?
(The range of a function is its set of *outputs,* all possible numbers that could result as an output value.)

- All real numbers greater than or equal to 0.
- $\{y : y \geq 0\}$
- $[0, \infty)$

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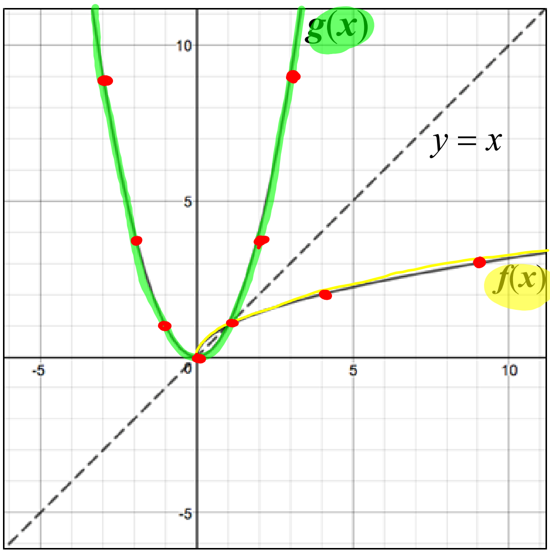
5. The graph of $f(x)=\sqrt{x}$, along with the graphs of $g(x)=x^2$ and $y=x$, are graphed below.

What do you notice about the graphs of $f(x)$ and $g(x)$?



Based on this observation, we can note that $f(x)=\sqrt{x}$ and $g(x)=x^2$ appear to be inverse functions. Specifically, we should note, it's only the right half of the $g(x)=x^2$ graph that is symmetric with the graph of $f(x)=\sqrt{x}$, so we need to use a domain restriction for $g(x)$: $x \geq 0$.

Thus, we can conclude that $f(x)=\sqrt{x}$ and $g(x)=x^2$ are indeed inverse functions.
Algebraic confirmation of this fact is given below:

$$f\left(g(x)\right)=f(x^2) = \sqrt{x^2} = x \qquad g\left(f(x)\right)=g(\sqrt{x}) = \left(\sqrt{x}\right)^2 = x$$

Note: Technically, $\sqrt{x^2} = |x|$. But since we use the domain restriction x ≥ 0, x will never be negative, so we can ignore the absolute value.

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Recall, in Quadratics 4b when we compared the vertex form of a quadratic function $g(x) = a(x-h)^2 + k$ to the parent function $f(x) = x^2$. When doing so, we saw how a, h, and k can be used to describe the transformations of the parent function.

Those same transformations also apply to the oter functions, including the square root function.

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Transformations of the Square Root Function

When comparing a function $g(x) = a\sqrt{x-h}+k$
to its parent function $f(x) = \sqrt{x}$:

- **Sign of *a*:**
  - If $a < 0$, then you state that the graph is "reflected across the x -axis."
  - If $a > 0$, then you state that there is "no reflection across the x -axis."

- **Value of *a*:**
  - If $|a| > 1$, then you state that the graph is "vertically <u>stretched</u> by a factor of $|a|$."
  - If $0 < |a| < 1$, then you state that the graph is "vertically <u>compressed</u> by a factor of $|a|$."
  - If $|a| = 1$, then you state that the graph has "no vertical stretch."

- **Value of *h*:**
  - If $h > 0$, then you state that the graph has a "horizontal shift h units right."
  - If $h < 0$, then you state that the graph has a "horizontal shift h units left."
  - If $h = 0$, then you state that there is "no horizontal shift."

- **Value of *k*:**
  - If $k > 0$, then you state that the graph has a "vertical shift k units up."
  - If $k < 0$, then you state that the graph has a "vertical shift k units down."
  - If $k = 0$, then you state that there is "no vertical shift."

<u>**Domain and Range**</u>

- **Domain:** Set the <u>**radicand**</u> (the part under the radical symbol) greater than or equal to zero.

- **Range:** If $a > 0$, then the range is $\{y : y \geq k\}$
  If $a < 0$, then the range is $\{y : y \leq k\}$

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Example 1: Given $g(x)=3\sqrt{x-2}+5$

a) Describe how the parent function $f(x)=\sqrt{x}$ is transformed.

b) State the domain and range of the function

c) Sketch the graph.

Solution:

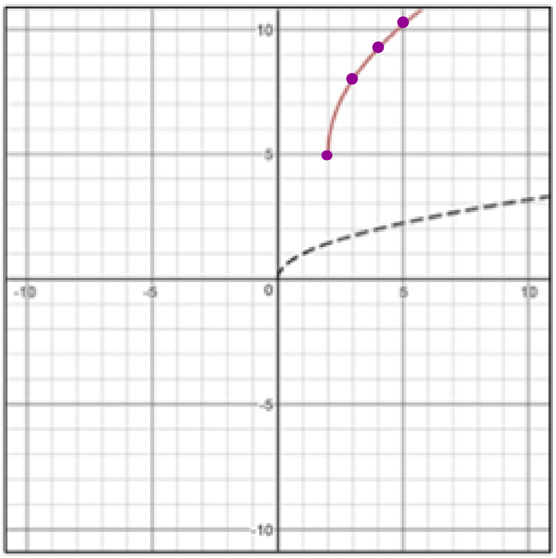
a) Describe how the parent function $f(x)=\sqrt{x}$ is transformed. a = 3, h = 2, k = 5

- No reflection across the x-axis
- Vertically stretched by a factor of 3
- Horizontal shift 2 unit right
- Vertical shift 5 units up

b) Domain: $\{x:\ x\geq 2\}$

$x-\cancel{2}\geq 0$
$+\cancel{2}\quad +2$
$x\geq 2$

Range: $\{y:y\geq 5\}$



c) Sketch -->

Vertex at (2,5) then plug in x values to the right.

x	y
2	5
3	8
4	9.2
5	10.2

Example 2: Given $g(x) = -\frac{1}{2}\sqrt{x+2}+3$

Solution:

$a = -\frac{1}{2}$, $h = -2$, $k = 3$

Reflected over x-axis.

Vertically compressed by a factor of $\frac{1}{2}$.

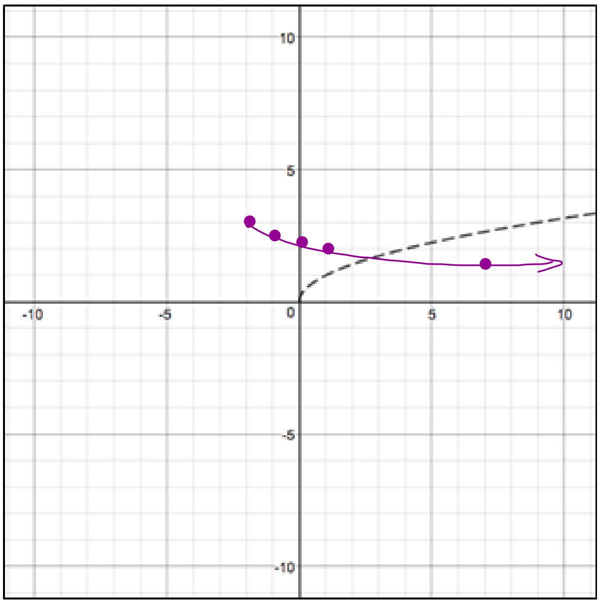
Shifts 2 units left.

Shifts 3 units up.

Domain: $\{x : x \geq -2\}$

$x+2 \geq 0$
$\;\;-2 \;\; -2$
$x \geq -2$

- All real numbers greater than or equal to -2.
- $[-2, \infty)$



Range: $\{y : y \leq 3\}$

- All real numbers less than or equal to 3.
- $(-\infty, 3]$

x	y
-2	3
-1	2.5
0	2.3
1	2.1
2	2

$f(-1) = -\frac{1}{2}\sqrt{(-1)+2}+3$

$= -\frac{1}{2}\sqrt{1}+3$

$= -\frac{1}{2}(1)+3$

$= -\frac{1}{2}+3$

$= 2.5$

Practice: For each function below:

a) Describe how the parent function $f(x) = \sqrt{x}$ is transformed.

b) State the domain and range of the function

c) Sketch the graph.

1) $g(x) = -2\sqrt{x-4} - 3$

Solution:

$a =$ -2, $h =$ 4, $k =$ -3

Reflected over x-axis

Vertically stretched by a factor of 2.

Shift 4 units right.

Shift 3 units down.

Domain: • $\{x : x \geq 4\}$

$x - 4 \geq 0$
$+4 \quad +4$
$x \geq 4$

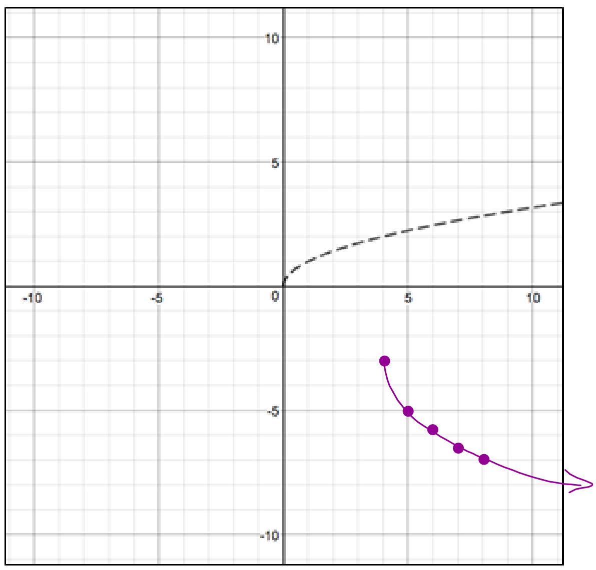
• All real numbers greater than or equal to 4.

• $[4, \infty)$

Range: • $\{y : y \leq -3\}$

• All real numbers less than or equal to -3.

• $(-\infty, -3]$



x	y
4	-3
5	-5
6	-5.8
7	-6.5
8	-7

2) $g(x) = 3\sqrt{x+3} - 7$

Solution:

$a =$ 3 , $h =$ -3 , $k =$ -7

No reflection over x-axis.

Vertically stretched by a factor of 3.

Shifts 3 units left.

Shifts 7 units down.

Domain: • $\{x : x \geq -3\}$

$x + \cancel{3} \geq 0$
$-\cancel{3} \quad -3$
$x \geq -3$

• All real numbers greater than or equal to -3.

• $[-3, \infty)$

Range: $\{y : y \geq -7\}$

x	y
-3	-7
-2	-4
-1	-2.8
0	-1.8
1	-1
6	2