Polynomials 4 - Symbolic Representation From a Graph

Standards: A-APR.2, A-APR.3, F-IF.4 **HW#8**

GLO: #3 Complex Thinker

Math Practice: Reason Abstractly & Quantitatively

Learning Target:

How can we create a symbolic representation of a polynomial using the graph?

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Determining a Function's Symbolic Representation from its Graphs

Example A: The graph of a polynomial function f(x) is shown below.

(erase to show)

a. First, notice there are x-intercepts/zeros at x = -1, 1, and 3. This tells us the factors of f(x) are : **(x+1)** , **(x - 1)** , and **(x - 3)** .

b. Notice the zero at x = -1 has **even** multiplicity because the graph **turns around/bounces there** .

Let's assume multiplicity = **2** .
So, **$(x + 1)^2$** is a factor of f(x).

c. Notice the zero at x = 1 has **odd** multiplicity because the graph **gets flat/swerves there** .

Let's assume multiplicity = **3** .
So, **$(x - 1)^3$** is a factor of f(x).

d. Reasonable guess is $f(x)=(x+1)^2(x-1)^3(x-3)$ (write in)
but because end behavior is ↙ ↘
we need to adjust it to:

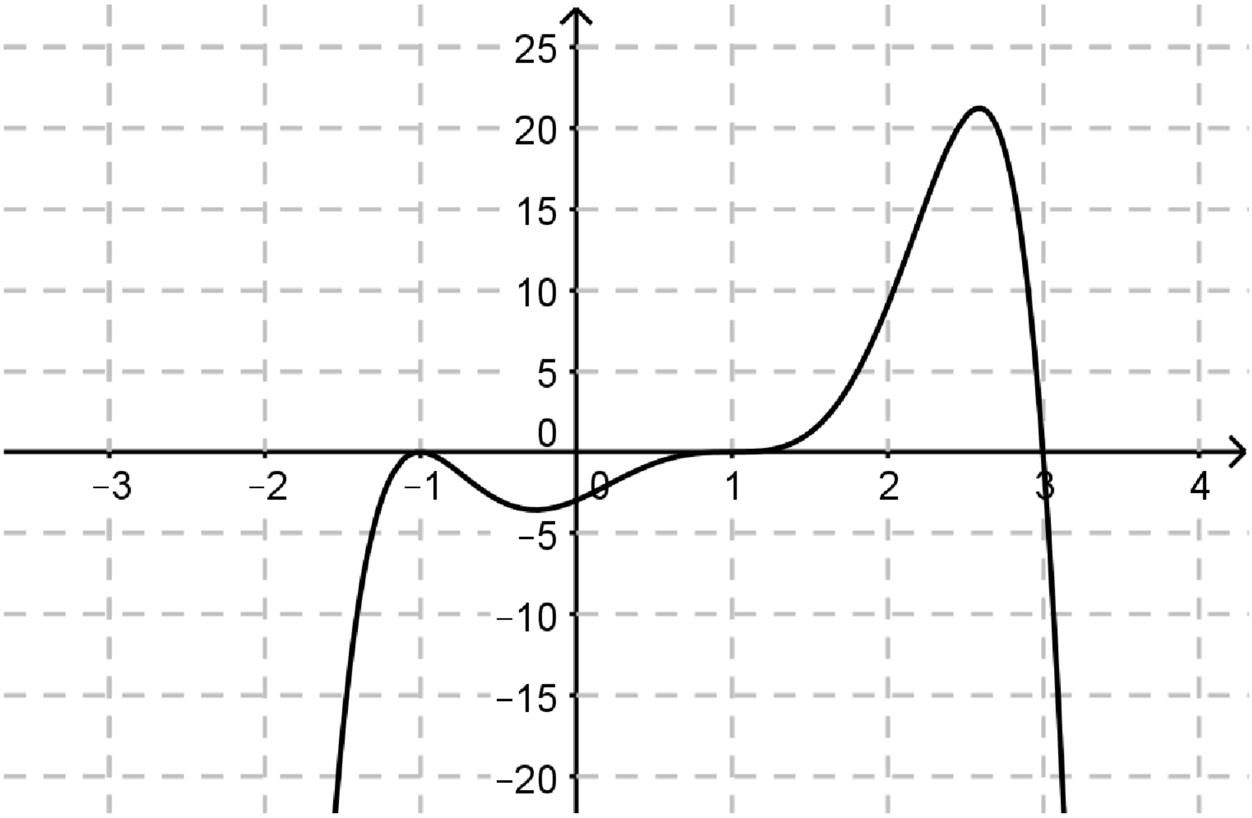
$$f(x)=-1(x+1)^2(x-1)^3(x-3)$$

e. Check the y-intercept: between 0 & -5

$$f(0)=-1(0+1)^2(0-1)^3(0-3)$$
$$=-1(1)^2(-1)^3(-3)$$
$$=+1(1)(+1)(-3)$$
$$f(0)=-3 \checkmark$$

(erase to show)

Example A:



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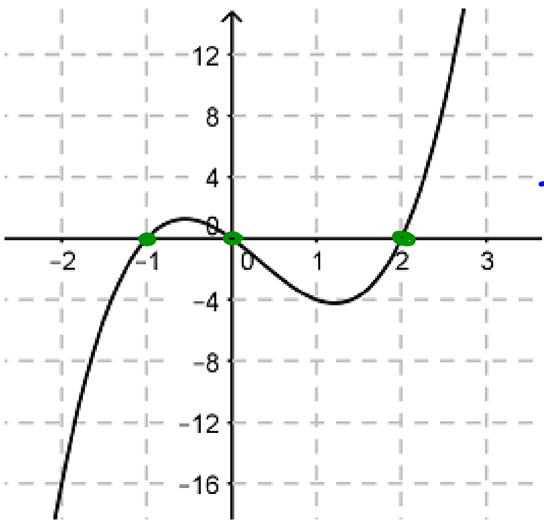
Example B: The graph of a polynomial function f(x) is shown below.



zeros: −1 , 0 , 2

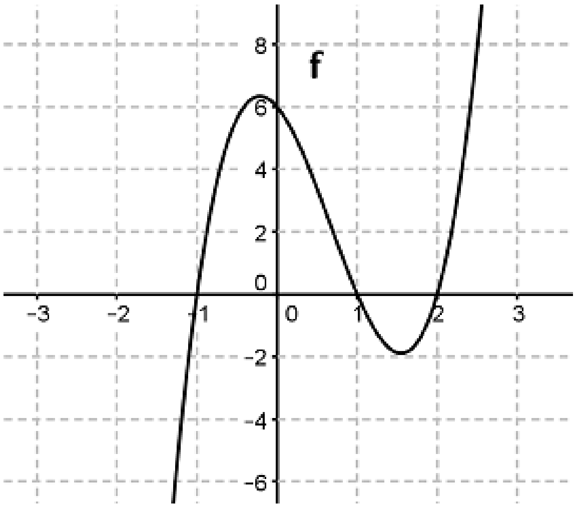
$(x+1)(x+0)(x-2)$

$\boxed{f(x) = x(x+1)(x-2)}$

D: 3

LC: +

Example C: The graph of a polynomial function, f(x) is shown below.



a. Since f has x–intercepts/ zeros at x = -1, 1, and 2, the factors of f are

$(x+1)$, $(x-1)$, $(x-2)$

b. Therefore, an initial guess for the symbolic representation of f is

$f(x) = (x+1)(x-1)(x-2)$

c. Unfortunately, the initial guess we just made is not quite right...

>According to this function, f(0)=2. y-int : (0, 2)

> However, according to the graph the y-intercept is (0,6), and thus, f(0) = 6. y-int: (0, 6)

Without changing our x-intercepts (i.e., the factors), adjust our initial guess so that f(0)=6:

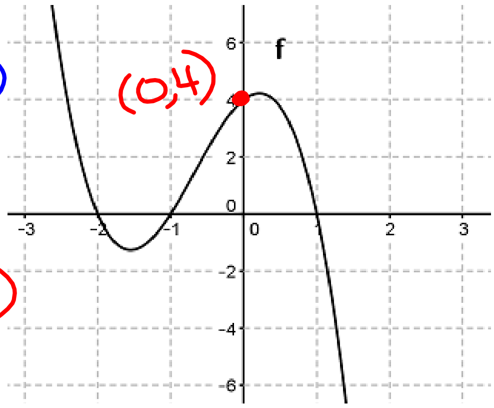
$f(x)=3(x+1)(x-1)(x-2)$

d. Check your symbolic representation to make sure all intercepts match the graph, and that the end-behavior matches that of the graph.

Practice: Determine the possible symbolic representation for each of the polynomial functions whose graphs are provided below.

- Check your answers to ensure the functions match the graphs at the intercepts and end-behavior.

1. f(x) = $-2(x+2)(x+1)(x-1)$



zeros: $-2, -1, 1$

$f(x)=(x+2)(x+1)(x-1)$

y-int: $(0,-2) \to (0,4)$

or $(0,4)$ x y

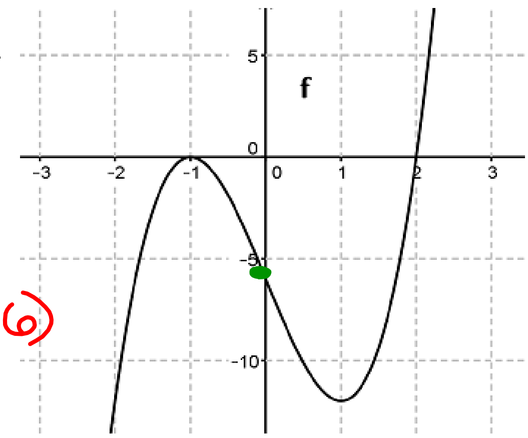
$4=a(0+2)(0+1)(0-1)$

$4=a(2)(1)(-1)$

$\frac{4}{-2}=\frac{a(\cancel{-2})}{\cancel{-2}}$

$\boxed{a=-2}$

2. f(x) = $3(x+1)^2(x-2)$



zeros: $-1, 2$

bounce (even mult.)

$f(x)=(x+1)^2(x-2)$

y-int: $(0,-2) \to (0,-6)$

or $(0,-6)$ x y

$-6=a(0+1)^2(0-2)$

$-6=a(1)^2(-2)$

$-6=a(1)(-2)$

$\frac{-6}{-2}=\frac{a(\cancel{-2})}{\cancel{-2}}$

$\boxed{a=3}$

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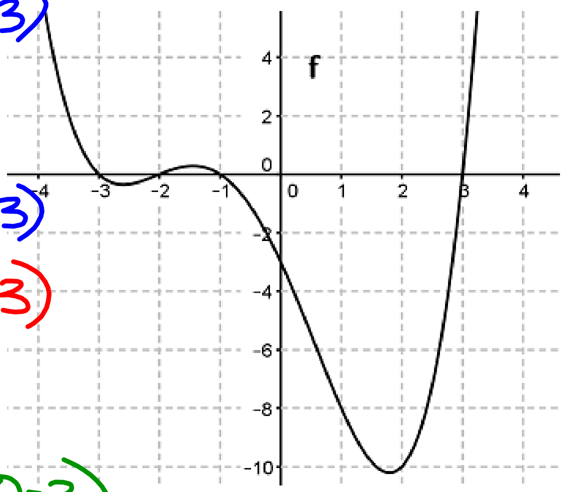
3. f(x) = $\frac{1}{6}(x+3)(x+2)(x+1)(x-3)$



zeros: −3, −2, −1, 3

$f(x)=(x+3)(x+2)(x+1)(x-3)$

y-int: $(0,-18) \rightarrow (0,-3)$

y-int: $(0,-3)$
x y

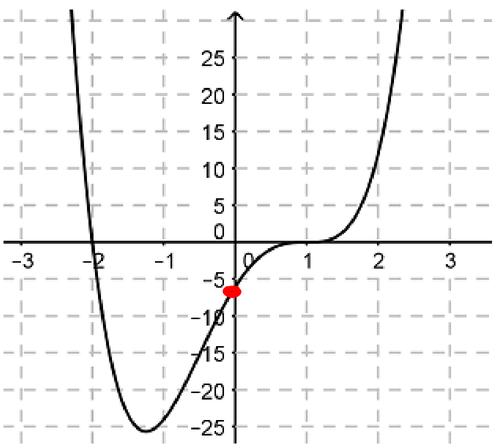
$-3=a(0+3)(0+2)(0+1)(0-3)$

$-3=a(3)(2)(1)(-3)$

$\frac{-3}{-18}=\frac{a\cancel{(-18)}}{\cancel{-18}}$

$\boxed{a=\frac{1}{6}}$

4. f(x) = $3(x+2)(x-1)^3$



zeros: −2, 1 ← swerve (odd mult)

$f(x)=(x+2)(x-1)^3$

y-int: $(0,-2) \rightarrow (0,-6)$

OR
y-int: $(0,-6)$
x y

$-6=a(0+2)(0-1)^3$

$-6=a(2)(-1)^3$

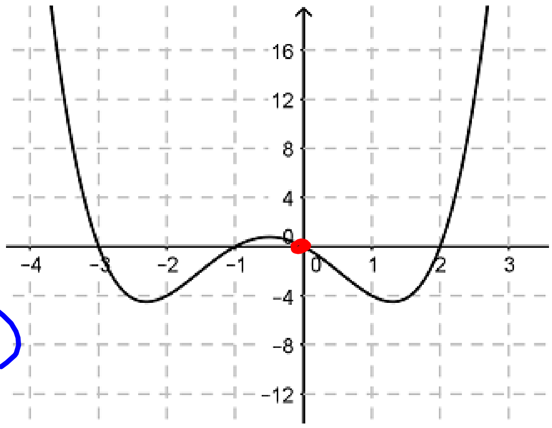
$-6=a(2)(-1)$

$\frac{-6}{-2}=\frac{a\cancel{(-2)}}{\cancel{-2}}$

$\boxed{a=3}$

(last slide)

5. $f(x) = x(x+3)(x+1)(x-2)$



zeros: $-3, -1, 0, 2$

$f(x) = (x+3)(x+1)(x+0)(x-2)$

y-int: $(0,0) \rightarrow (0,0)$ ✓