Polynomial 3a - Zeros & Repeated Zeros

Standards: A-APR.3, F-IF.7c **HW#6**

GLO: #3 Complex Thinker

Math Practice: #2 Reason Abstractly & Quantitatively

Learning Target

-What do we need to put together a polynomial graph?

-Describe the ways a graph behaves when zeros repeat.

Oct 30-9:18 PM

Warm up

1. Which of the following functions are polynomials?

$a(x) = (x-5)(x+3)$ $\quad$ $d(x) = x^5 + 3x^4 - 8x^2 + 2$

$b(x) = \dfrac{x+1}{2x-7}$ exp not whole #s $\quad$ $e(x) = 2x - 1$

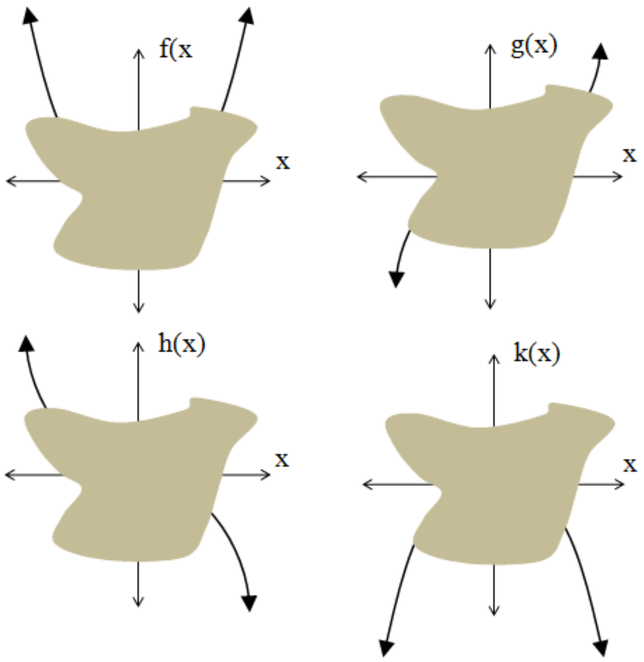
$c(x) = 2 \cdot 3^x$ exp not always whole #s $\quad$ $f(x) = \sqrt{7x-5}$ exp not whole #s

Jan 8-11:01 PM

2. Kimo just sat down at the table with his math homework when his mom asked him to feed his baby brother. Unfortunately for Kimo, poi was on the menu – what a mess!

"Oh no," said Kimo, "There is poi on my homework. My graphs are ruined. How can I match the graphs to the functions when I can't see all of the graphs?"

Please help Kimo match each graph to its equation.



$-2x^3$

$h(x) = -2x(x+4)(x-1)$

$f(x) = 3x^4 - 9x^3 + 2x^2 - 5x + 1$

$k(x) = -(x+3)^3(x-1)$ $-x^4$

$g(x) = x^3 - 100x$

From Quadratic to Polynomial Functions:

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Zeros (**x - intercepts**) are key to graphing quadratics and polynomials.

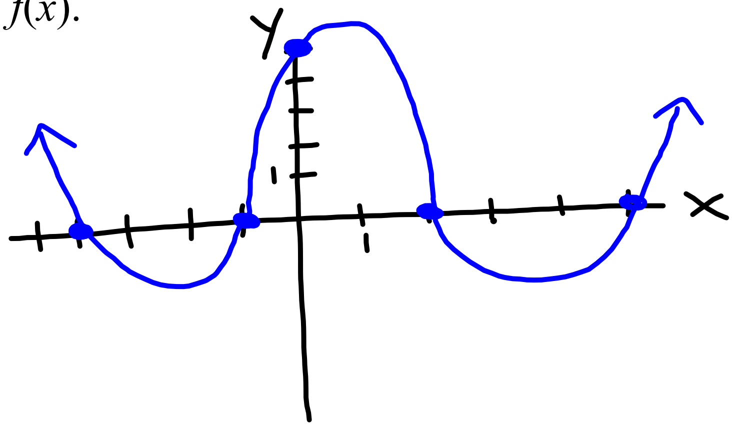
For quadratics, we find zeros by factoring, square rooting, quadratic formula, etc…Not so easy for polynomials. So first, lets get good at graphing if we know the zeros.

Example 1:

Suppose $f(x)$ is a 4th degree polynomial with positive leading coefficient, a constant term equal to 5, and zeros at $x = -4, -1, 2, \& 5$. Sketch a possible graph of $f(x)$.

max of 3 turns

y-int: (0,5)



There are actually infinitely many polynomials that satisfy the same given information...so our goal is only to identify the general behavior of $f(x)$.

Basic Guidelines for Sketching Polynomial Functions:

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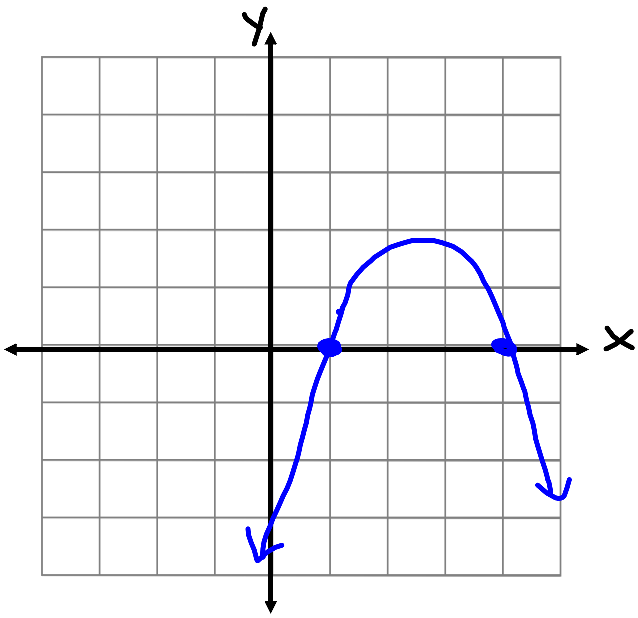
- Use the degree and leading coefficient to determine the **end behavior** .
- Use the constant to draw a point at the **y-intercept** .
- Use the zeros to draw point(s) at the **x-intercept(s)** .
- Sketch the general shape.

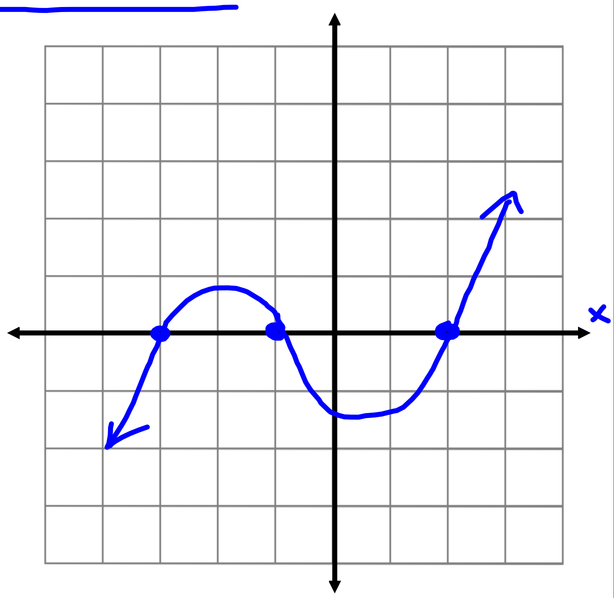
Practice: Sketch a possible graph for each of the following polynomial functions based on the given information.

1. f is a polynomial function such that
- its degree is even
- its leading coefficient is negative
- its constant term is negative
- it has zeros only at x = 1 and x = 4

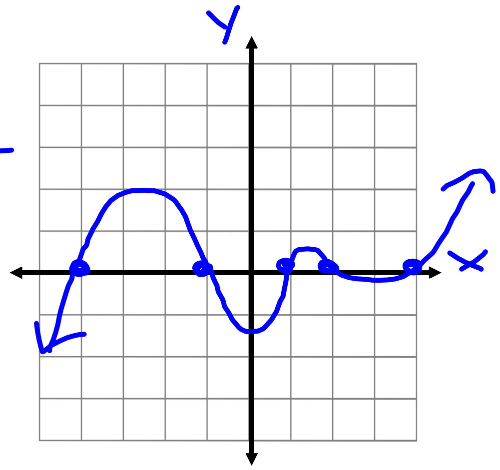
y-int: (0, -#)



2. f is a polynomial function such that
- its degree is odd
- its leading coefficient is positive
- its constant term is negative (0, -#)
- it has zeros only at $x = -3$, $x = -1$ and $x = 2$



3. f is a polynomial function such that
- its degree is odd
- its leading coefficient is positive
- its constant term is negative (0, -#)
- it has zeros only at $x = -4$, $x = -1$, $x = 1$
$x = 2$ and $x = 4$



Repeated Zeros

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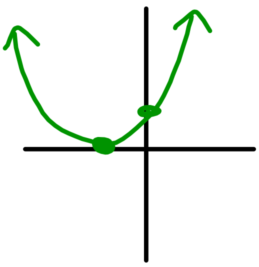
Repeated Zero – Occurs when a **factor** of a polynomial is repeated.

Example: $f(x) = x^2 + 2x + 1$ y-int: (0,1)

Graph:



$x + \cancel{1} = 0$
$\cancel{-1} \quad -1$
$x = -1$

$0 = (x+1)(x+1)$

$= (x+1)^2$

$(x+1)$ is a repeated factor

$x = -1$

"bounce"

So the graph touches or **turns around** or changes direction at the zero.

Another way to describe this repeated zero is to say f(x) has a zero at x = –1 with **multiplicity** 2.

Example:

$g(x) = x^3$

$= x \cdot x \cdot x$

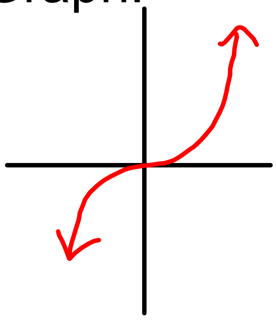
$= (x-0)(x-0)(x-0)$

$= (x-0)^3$

$(x-0)$ is a repeated factor

$x = 0$

Graph:



So the graph **"gets flat"** at the zero.
(or swerves)

Another way to describe this repeated zero is to say g(x) has a zero at x = 0 with **multiplicity** 3.

Jan 18-2:44 PM

Zeros with __**even**__ **multiplicity will turn around** (bounce)

Zeros with __**odd**__ **multiplicity will get flat** (swerve)

Remember: These are the general shapes of possible graphs, so no scale on y-axis since we really don't know y-intercepts, heights, etc.

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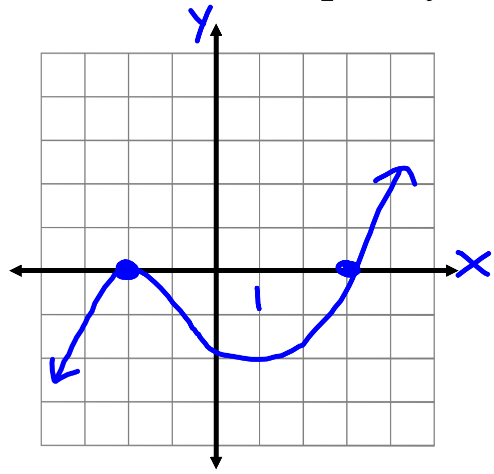
Example 2:

Provide a possible graph of a polynomial function satisfying:

- Odd degree
- Positive leading coefficient
- Zeros only at $x = -2$ & 3, where $x = -2$ has multiplicity 2.

bounce

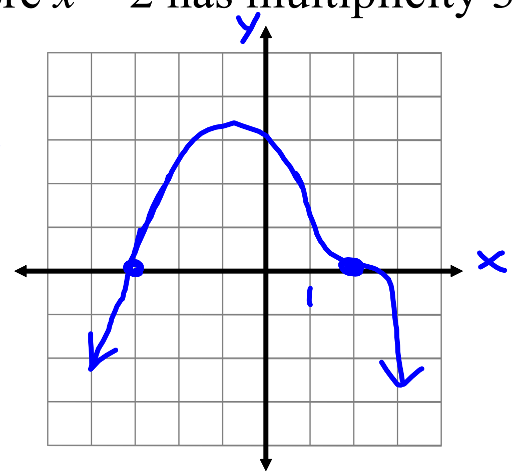


Example 3:

Provide a possible graph of a polynomial function satisfying:

- Even degree
- Negative leading coefficient
- Zeros only at $x = -3$ & 2, where $x = 2$ has multiplicity 3

swerve



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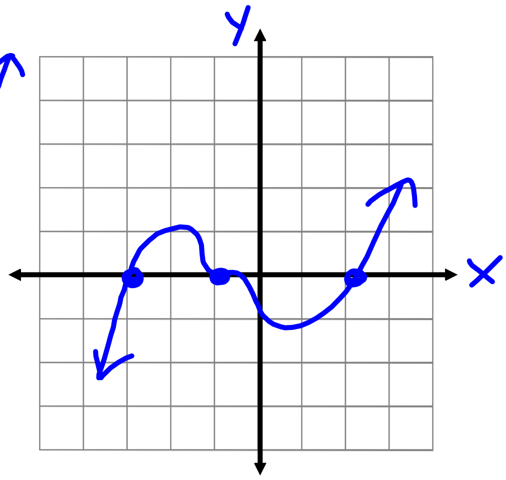
Practice: Sketch a possible graph for each of the following polynomial functions based on the given information.

4. f is a polynomial function such that
- its degree is odd
- its leading coefficient is positive
- it has zeros only at $x = -3$, $x = -1$ and $x = 2$, where $x = -1$ has multiplicity 3

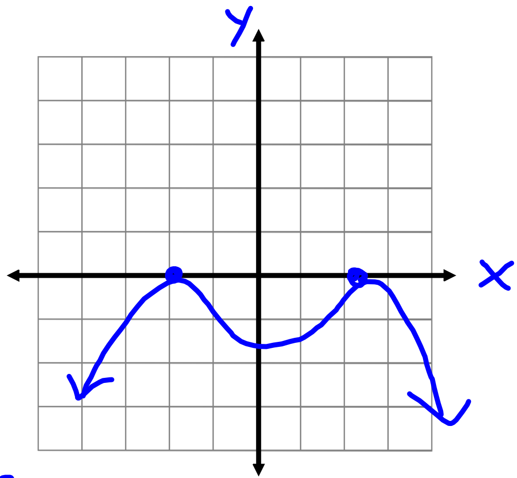
swerve



5. f is a polynomial function such that
- its degree is even
- its leading coefficient is negative
- it has zeros only at $x = -2$ and $x = 2$, where $x = -2$ has multiplicity 2 and $x = 2$ has multiplicity 4.

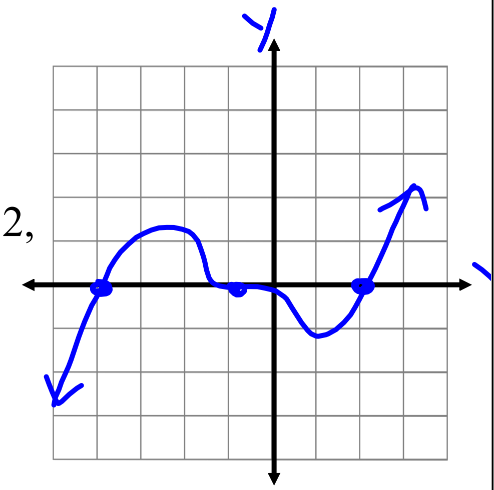
bounce

bounce



6. f is a polynomial function such that
- its degree is odd
- its leading coefficient is positive
- it has zeros only at $x = -4$, $x = -1$ and $x = 2$, where $x = -1$ has multiplicity 3

swerve



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