## Functions 5b - Composition Continued

Standards: F-BF.1, F-BF.1c HW\#4: Func 5b \#1-9

GLO: \#3 Complex thinker
Math Practice: \#7 Look for and make use of structure
Learning Target:
How do you do compositions of functions in different forms?

Last time we practiced composing functions that are given to us. Many times, though, we need to identify the individual functions that comprise a composition. For example, given the function $h(x)=3(x-5)^{3}-4$ we can write $h(x)=f(g(x))$ with the inside function $g(x)=x-5$ and the outside function $f(x)=3 x^{3}-4$

1. Below, you are given a number of functions, each labeled $h$. Your job for each $h$ is to identity functions $f$ and $g$ so that $h(x)=f(g(x))$
a) $h(x)=\frac{1}{4 x-5}$

$$
\begin{aligned}
& f(x)=\frac{\frac{1}{x}}{g(x)=4 x-5}
\end{aligned}
$$

b) $h(x)=\sqrt{3 x^{2}-5 x+2}$

$$
\begin{aligned}
& f(x)=\frac{\sqrt{x}}{g(x)=3 x^{2}-5 x+2}
\end{aligned}
$$

c) $h(x)=|2 x-5|$

$$
\begin{aligned}
& f(x)=\frac{|x|}{g(x)=2 x-5}
\end{aligned}
$$

d) $h(x)=2(3 x-5)^{2}-7(3 x-5)+2$

$$
\begin{aligned}
& f(x)=2 x^{2}-7 x+2 \\
& g(x)=3 x-5
\end{aligned}
$$

We can also compose functions if they are given to us in different representations.
Below, you are given the graph of $f(x)$, a chart of values for $g(x)$, and the symbolic form for $h(x)$.


| $\boldsymbol{x}$ | $g(x)$ |
| :---: | :---: |
| -2 | 3 |
| -1 | 0 |
| 0 | 2 |
| 1 | 3 |
| 2 | -1 |
| 3 | -2 |
| 4 | 1 |

$$
h(x)=\frac{1}{2} x^{2}
$$

We can use these three functions (with different representations) to compute specific values for composite functions. For example, if we want to compute $\mathbf{f}(\mathbf{g}(\mathbf{2}))$, we do the following, working from the inside out:

Evaluating $g$ at 2 we get: $g(2)=-1$
So plugging in $g(2)=-1$ into $f(g(2))$, we get

$$
f(g(2))=f(-1)=0
$$

So $f(g(2))=0$.
Schematically, we can see this below. To compute the composite value $f(g(2))$, we first plug in 2 into the function g , which outputs the middle -1 . Then we plug in $g(2)=-1$ into $f$ to get out a 0 .



Evaluate the following composition of functions at the given values.
2)

$$
\begin{aligned}
& f(-1)=0 \\
& g(0)=2
\end{aligned}
$$

4) $g(h(-2))=-1$

$$
\begin{aligned}
& h(-2)=\frac{1}{2}(-2)^{2} \\
& h(-2)=2 \\
& g(2)=-1
\end{aligned}
$$

6) $f(f(2))=-2$

$$
\begin{aligned}
& f(2)=0 \\
& f(0)=-2
\end{aligned}
$$

3) $f(g(-2))=4$

$$
g(-2)=3
$$

$$
f(3)=4
$$

5) $h(h(4))=32$

$$
\begin{aligned}
& h(4)=\frac{1}{2}(4)^{2} \\
& h(4)=8 \\
& h(8)=\frac{1}{2}(8)^{2} \\
& h(8)=32
\end{aligned}
$$

7) $g(g(0))=-1$
$g(0)=2$
$g(2)=-1$

The composition of functions is not limited to just two functions. We can compose three functions (or any number of functions), as long as we work from the inside out (as we did above), and as long as we do not encounter domain conflicts along the way. Evaluate the following composition of functions:
8) $h(g(f(0)))=4$.
$f(0)=-2$
$g(-2)=3$
$h(3)=\frac{1}{2}(3)^{2}$
$=4.5$
9) $g(g(h(2)))=$

$$
h(2)=\frac{1}{2}(2)^{2}=2
$$

$g(2)=-1$
$g(-1)=0$

