Polynomials 2a - End Behavior

Standards: A-SSE.1.a, A-SSE.1.b, F-IF.7c

HW#4

#1-3

GLO: #3 Complex Thinker

Math Practice:
Look for and express regularity in repeated reasoning.

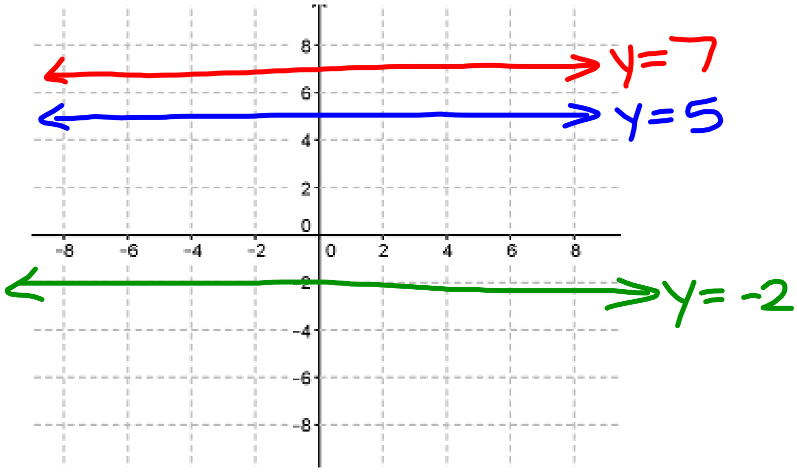
Learning Target:
What characteristics tell you about the end behavior?

Set #1

$y = 5$

$y = -2$

$y = 7$



The graphs of polynomial functions of degree $\boxed{0}$ are $\boxed{\text{horizontal}}$

Set #2

$m = \frac{-2}{3}$
$b = 1$

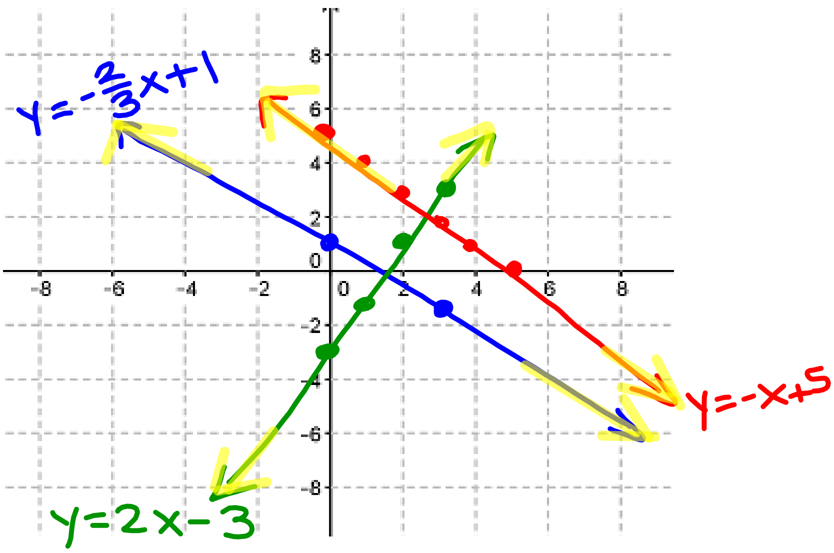
$y = -\frac{2}{3}x + 1$

$m = \frac{2}{1}$
$b = -3$

$y = 2x - 3$

$m = \frac{-1}{1}$
$b = 5$

$y = -x + 5$



The graphs of polynomial functions of degree $\boxed{1}$ are $\boxed{\text{linear}}$

Do the left and right ends of each function head the same direction or opposite directions?

If the leading coefficient is $\boxed{\text{positive}}$, then the graph increases to the right.

If the leading coefficient is $\boxed{\text{negative}}$, then the graph decreases to the right.

Set #3

y-int: (0,5) concave up

vertex: (0,5)

$y = x^2 + 5$

vertex: (2,3) concave down

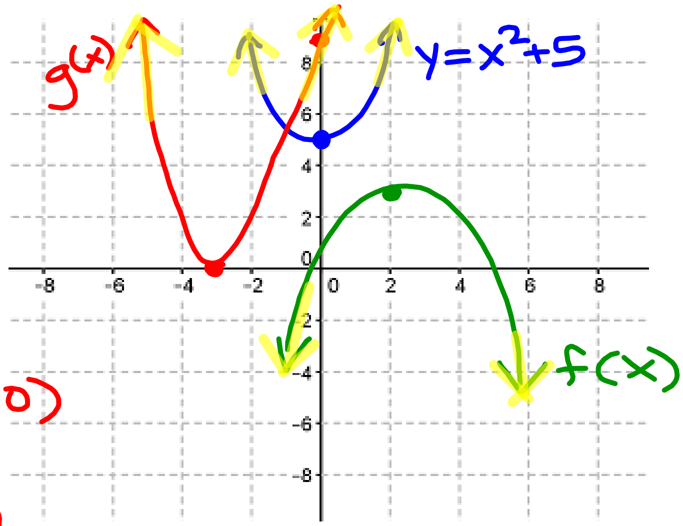
$f(x) = -(x-2)^2 + 3$

y-int: (0,9) x-int: (-3,0)

concave up $g(x) = x^2 + 6x + 9$

$= (x+3)(x+3)$

vertex: (-3,0)



The graphs of polynomial functions of degree 2 **are** parabolas

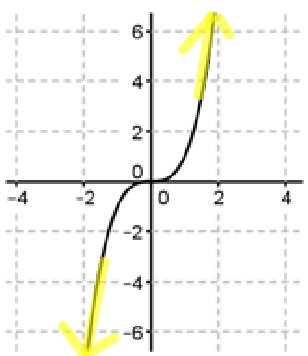
Do the left and right ends of each function head the same direction or opposite directions?

If the leading coefficient is positive **, then the graph opens up.**

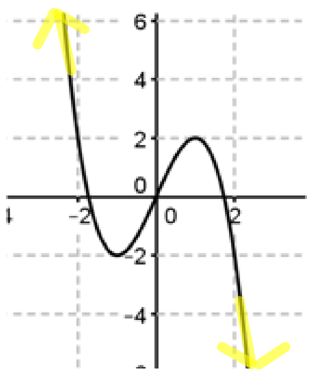
If the leading coefficient is negative **, then the graph opens down.**

Set#4

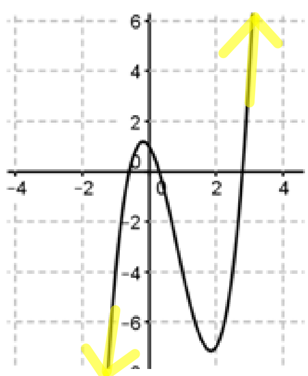
$f(x) = x^3$



$g(x) = -x^3 + 3x$



$h(x) = 2x^3 - 5x^2 - 2x + 1$



The graphs of polynomial functions of degree 3 **are** curvy lines

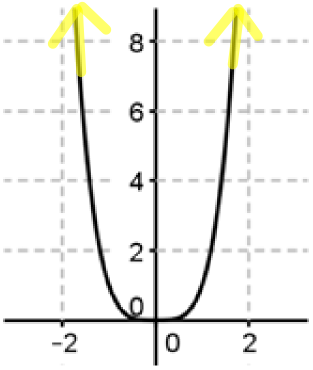
Do the left and right ends of each function head the same direction or opposite directions?

If the leading coefficient is positive **, then the graph increases to the right.**

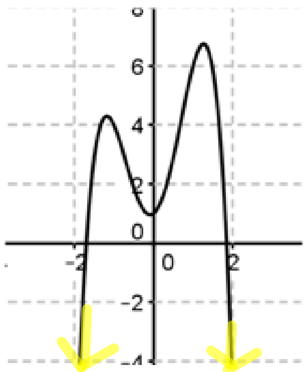
If the leading coefficient is negative **, then the graph decreases to the right.**

Set #5

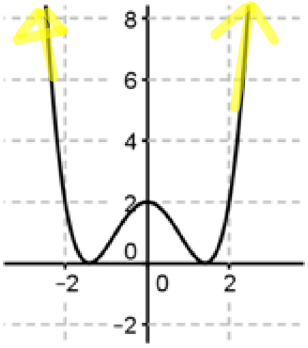
$y = x^4$



$y = -2x^4 + 6x^2 + x + 1$



$y = 0.5x^4 - 2x^2 + 2$



The graphs of polynomial functions of degree 4 **are** curvy lines

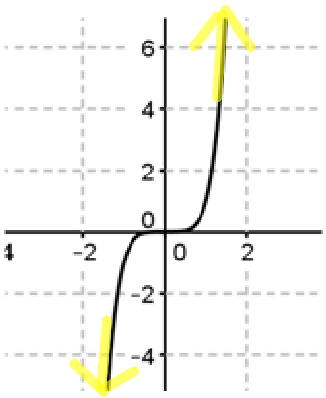
Do the left and right ends of each function head the same direction or opposite directions?

If the leading coefficient is positive **, then the graph opens up.**

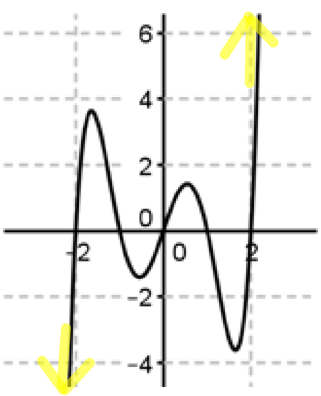
If the leading coefficient is negative **, then the graph opens down.**

Set #6

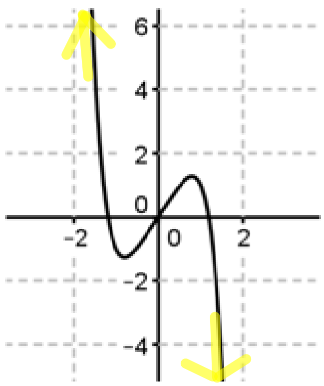
$y = x^5$



$y = x^5 - 5x^3 + 4x$



$y = -x^5 + 2x$



The graphs of polynomial functions of degree 5 **are** curvy lines

Do the left and right ends of each function head the same direction or opposite directions?

If the leading coefficient is positive **, then the graph increases to the right.**

If the leading coefficient is negative **, then the graph decreases to the right.**

Summary:

As you look back over your graphs for your what can you conclude about the left and right end behavior (i.e. which way do the two ends of the function point)?

$+x^{even}$ ↖ ↗ $-x^{even}$ ↙ ↘

$+x^{odd}$ ↙ ↗ $-x^{odd}$ ↖ ↘

Why do you think that all cubic polynomials (3rd degree) have similar end behavior regardless of the terms in the polynomial other than the leading 3rd degree term?

Because x^3 has a bigger effect than something times x.

Why is it that even degree polynomials behave differently from odd degree polynomials?

Even degrees mimic a parabola, while odd degrees mimic a linear graph.

The number at the end of the function's symbolic representation (the constant term) identifies what part of the graph? y-intercept. Look back at your graphs to see for yourself.

Notice that a cubic polynomial (a 3rd degree polynomial) can have zero "turns" as is the case with $f(x) = x^3$ or it can have two turns as is the case with $f(x) = -x^3 + 3x$. A "turn" here refers to a point where the function changes direction from increasing to decreasing. In mathematical terms it is referred to as a relative maximum or minimum. For example, a parabola has a turn at its vertex. Is it possible for a cubic polynomial to have exactly one turn? If so, graph one. If not, explain why it is not possible.

ex:

Identify degree, LC, & y-int. Then, graph end behavior.

$m(x) = -2x^7 + 4x^3 + 17x^2 + 5x - 4$

D: 7 (opp direction)

LC: -2 (ending down)

y-int: (0,-4)