Module 7a: Intro to Dilations

Math Practice(s):

-Make sense of problems & persevere in solving them.

-Use appropriate tools strategically.

Learning Target(s):

-Understand that dilations create proportional sides and preserve corresponding angle measures.

Homework:

HW#9: 7a #1-6

Aug 8-12:20 PM

Warm-up

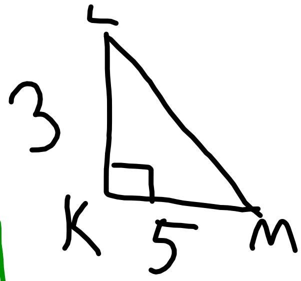
1. Draw ΔKLM such that $\overline{KL} \perp \overline{KM}$, where $KL = 3$ units and $KM = 5$ units. Then, determine LM.

$3^2+5^2=c^2$

$9+25=c^2$

$\sqrt{c^{\cancel{2}}}=\sqrt{34}$

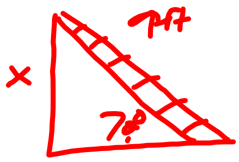
$\boxed{LM=\sqrt{34} \text{ units} \approx 5.831 \text{ units}}$



2. A ladder is placed against a wall so that it forms a $70°$ angle with the ground (with the wall being perpendicular to the ground). If the ladder is 9 feet long, how high up the wall does the ladder reach? (**Draw a diagram** to represent ~~the situation~~ the situation described.)

$\cancel{9}(\sin 70°)=\left(\frac{x}{\cancel{9}}\right)\cancel{9}$

$x=8.457 \text{ ft}$



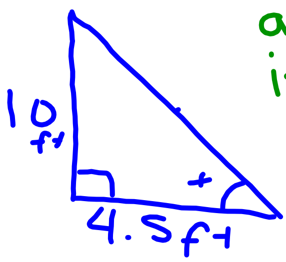
The ladder reaches about 8.457 feet up the wall.

3. A different ladder is placed against a wall so that the bottom of the ladder is on the ground 4.5 feet away from the bottom of the wall and the top of the ladder touches the wall at a height of 10 feet. (with the wall being perpendicular to the ground). What is the measure of the acute angle that is formed between the ladder and the ground? (**Draw a diagram** to represent the situation the situation described.)

$\tan\theta=\frac{10}{4.5}$

$\tan^{-1}\left(\frac{10}{4.5}\right)=\theta$

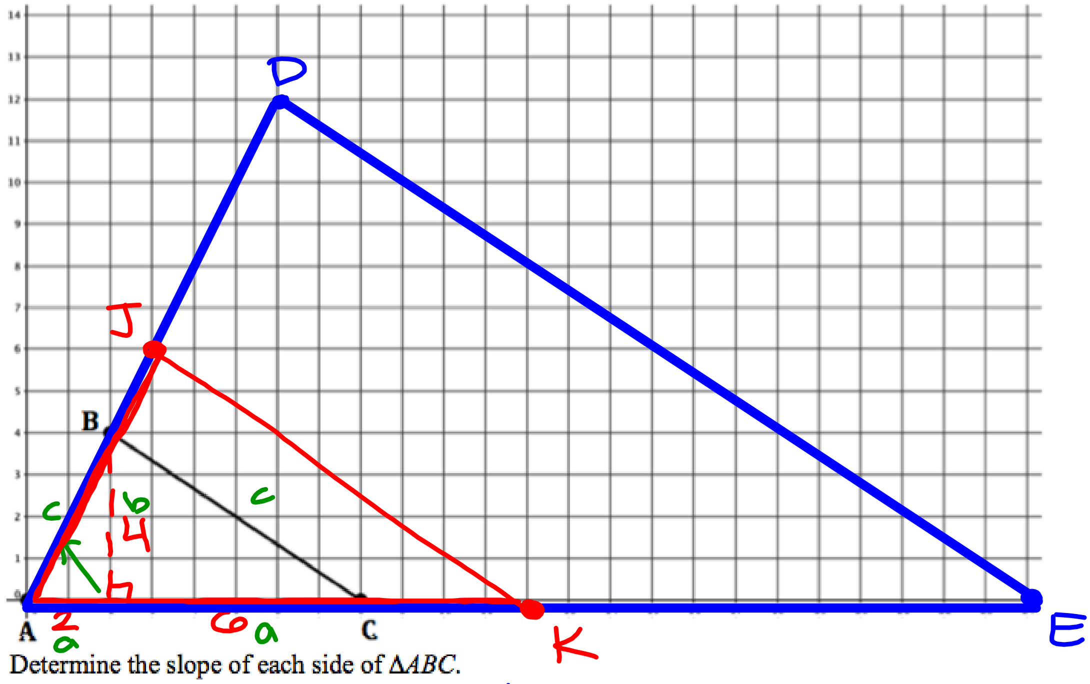
$\theta \approx 66°$



The measure of the acute angle that is formed between the ladder & the ground is about 66°.

Exploring Dilations

4. In the coordinate plane below, the scale used on both axes is 1 unit. ΔABC has vertices at the origin, (2, 4) and (8, 0).



A. Determine the slope of each side of ΔABC.

$m_{AB} = \frac{4}{2} = 2$ $m_{BC} = \frac{-4}{6} = -\frac{2}{3}$ $m_{AC} = \frac{0}{8} = 0$

B. Determine the length of each side of ΔABC. (Hint: for two of the sides, draw the altitude of ΔABC and then use the Pythagorean Theorem.)

$a^2+b^2=c^2$
$2^2+4^2=AB^2$
$\sqrt{AB^2}=\sqrt{20}$
$\boxed{AB=2\sqrt{5}\text{ units}}$

$a^2+b^2=c^2$
$6^2+4^2=BC^2$
$\sqrt{BC^2}=\sqrt{52}$
$\boxed{BC=2\sqrt{13}\text{ units}}$

$\boxed{AC=8\text{ units}}$

C. In the same coordinate plane, dilate ΔABC about the origin, creating ΔADE, such that $\Delta ADE \sim \Delta ABC$ and each side of ΔADE is 3 times longer (has a scale factor of 3) than its corresponding side in ΔABC. In your figure, label the length of each side of ΔADE.

D. Without doing any computations, state the slope of each side of ΔADE and explain how you determined your answers.

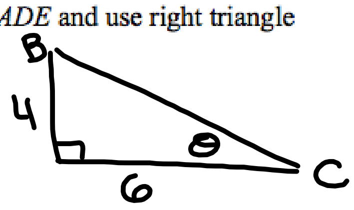
$m_{AD} = m_{AB} = 2$

$m_{DE} = m_{BC} = -\frac{2}{3}$

$m_{AE} = m_{AC} = 0$

\# Corresponding lines will have the same slopes to keep the corresponding $\cong$ angles in tact.

E. Determine the measure of $\angle ACB$. (Hint: draw the altitude of ΔADE and use right triangle trigonometry.)



$\tan \theta = \frac{4}{6}$

$\tan^{-1}\left(\frac{4}{6}\right) = \theta$

$\boxed{m\angle ACB = 34^\circ}$

F. Without doing any computations, state the measure of $\angle E$ and explain how you determined your answer.

$m\angle E = 34^\circ$, since $\Delta ADE \sim \Delta ABC$, their corresponding angles must be congruent.

G. In the same coordinate plane, dilate ΔADE about the origin using a scale factor of $\frac{1}{2}$ to form ΔAJK.

H. Determine the area of ΔABC, ΔADE, and ΔAJK.

ΔABC:
$b = 8$
$h = 4$
$A = \frac{1}{2}(8)(4)$
$\boxed{A = 16 \text{ units}^2}$

ΔADE: (sim ratio: $\frac{3}{1}$, area ratio: $\frac{9}{1}$)
$A = 9 \cdot A_{\Delta ABC}$
$= 9 \cdot 16$
$\boxed{A = 144 \text{ units}^2}$

ΔAJK (sim ratio: $\frac{3}{2}$, area ratio: $\frac{9}{4}$)

ΔAJK: (sim ratio: $\frac{1}{2}$, area ratio: $\frac{1}{4}$)
$A = \frac{1}{4} \cdot A_{\Delta ADE}$
$A = \frac{1}{4} \cdot 144$
$\boxed{A = 36 \text{ units}^2}$

(erase to show)

Scale Factor of a Dilation (#VOC): the number that describes the size change from an original figure to its image.

*ΔADE is a dilation about the origin of ΔABC with a **scale factor** of 3.

*ΔAJK is a dilation about the origin of ΔABC with a **scale factor** of 1.5 or $\frac{3}{2}$.

ΔAJK is a dilation about the origin of ΔADE with a scale factor of 0.5 or $\frac{1}{2}$.

Nov 21-11:18 AM