Module 4d: Special Right Triangles

Math Practice(s):

-Look for & express regularity in repeated reasoning.

Learning Target(s):

-Discover, understand, & apply the relationship between the length of the sides of special right triangles: "45-45-90" & "30-60-90".

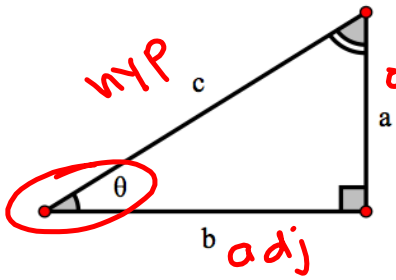
Homework:

HW#4: 4d #1-6

Aug 8-12:20 PM

Warm-up

1. For the right triangle below, write 3 equations (each using a different trigonometric ratio) to represent the relationship between the value of θ and the lengths of the sides of the triangle.

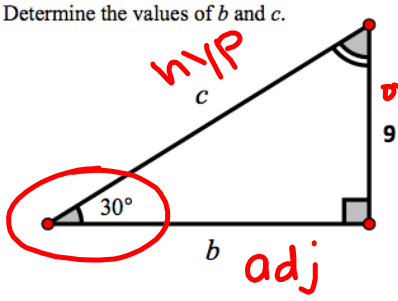


$\sin\theta = \frac{a}{c}$

$\tan\theta = \frac{a}{b}$

$\cos\theta = \frac{b}{c}$

2. Determine the values of b and c.



$c(\sin 30^\circ) = \left(\frac{9}{c}\right)c \qquad b(\tan 30^\circ) = \left(\frac{9}{b}\right)b$

$\frac{c\cdot\sin 30^\circ}{\sin 30^\circ} = \frac{9}{\sin 30^\circ} \qquad \frac{b\cdot\tan 30^\circ}{\tan 30^\circ} = \frac{9}{\tan 30^\circ}$

$c = 18$ units

$9^2 + b^2 = 18^2$

$81 + b^2 = 324$

$-81 \qquad -81$

$\sqrt{b^2} = \sqrt{243}$

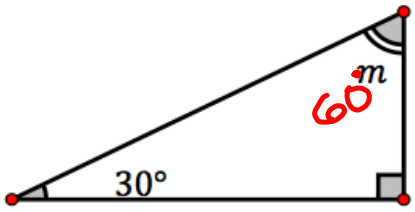
$\sqrt{81}\cdot\sqrt{3}$

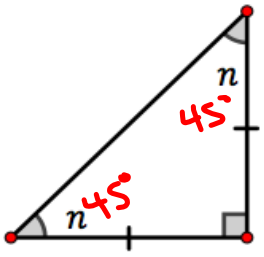
$b = 9\sqrt{3}$

$b = 15.588$ units

Two special right triangles

Determine the values of m and n in the triangles below.





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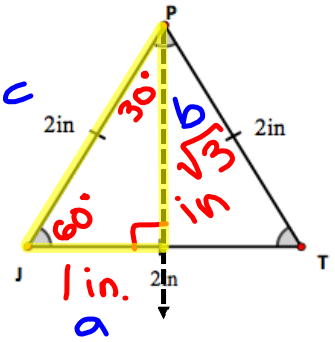
- The triangle on the left is called a "30 - 60 - 90" triangle.
- The triangle on the right is called a "45 - 45 - 90" triangle.

Investigation 1: The relationship between the lengths of the sides of a "30 – 60 – 90" triangle.

The figure to the right is an equilateral triangle.

- Draw the perpendicular bisector from vertex P to $\overline{JT}$.
- You will now see 2 triangles inside of the original triangle. Draw one of the triangles in the space to the right and label the measure of all 3 sides.



c
b
$\sqrt{3}$ in
a

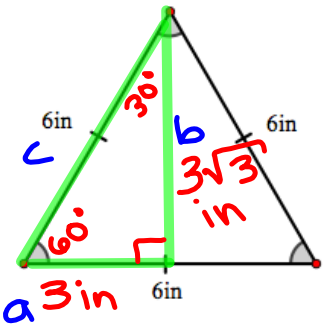
$1^2 + b^2 = 2^2$

$1 + b^2 = 4$

$-1 \quad -1$

$\sqrt{b^2} = \sqrt{3}$

$b = \sqrt{3}$ in



c
b
$3\sqrt{3}$ in
a

$3^2 + b^2 = 6^2$

$9 + b^2 = 36$

$-9 \quad -9$

$\sqrt{b^2} = \sqrt{27}$

$\sqrt{9} \cdot \sqrt{3}$

$b = 3\sqrt{3}$ in

Reflect and analyze: discuss the following with the partner and then answer each question.

- What do you notice about the ratio between the hypotenuse and the shortest leg for each of the three triangles you investigated?

$\frac{hyp}{SL} = \frac{2}{1}$

- If the shortest leg in a 30° - 60° - 90° triangle is 15 feet, how long is the hypotenuse?

30 ft

- What do you notice about the ratio between the longer leg and the shorter leg each of the three triangles you investigated?

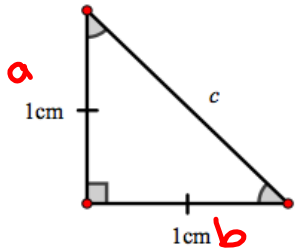
$\frac{LL}{SL} = \frac{3\sqrt{3}}{3} = \frac{1\sqrt{3}}{1} = \frac{\sqrt{3}}{1}$

Oct 25-2:46 PM

Investigation 2: The relationship between the lengths of the sides of a "45 – 45 – 90" triangle.

For all three isosceles right triangles below,

- Label the measure of the two acute angles of the triangle; then,
- determine the length of the hypotenuse.
  - For the length of the hypotenuse, state the exact measure (i.e., express your answer in radical form such that there are no perfect square factors under the radical).

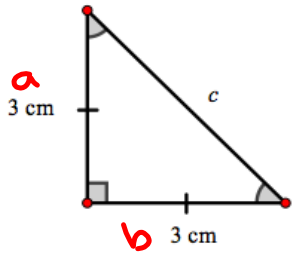


$1^2 + 1^2 = c^2$

$1 + 1 = c^2$

$\sqrt{2} = \sqrt{c^2}$

$c = \sqrt{2}$ cm



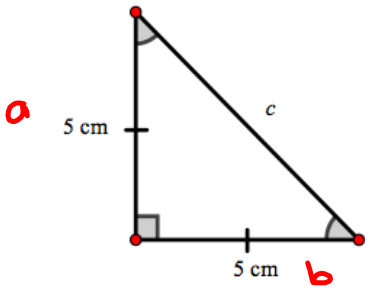
$3^2 + 3^2 = c^2$

$9 + 9 = c^2$

$\sqrt{18} = \sqrt{c^2}$

$\sqrt{9} \cdot \sqrt{2}$

$c = 3\sqrt{2}$ cm



$5^2 + 5^2 = c^2$

$25 + 25 = c^2$

$\sqrt{50} = \sqrt{c^2}$

$\sqrt{25} \cdot \sqrt{2}$

$c = 5\sqrt{2}$ cm

Reflect and analyze: discuss the following with the partner and then answer each question.

- What is the relationship between the leg measurement and the hypotenuse of isosceles right triangles? $hyp = leg \cdot \sqrt{2}$
- If the legs of an isosceles right triangle are 25 in, how long is the hypotenuse?

$25\sqrt{2}$ in

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Summarize the main ideas we just discovered:

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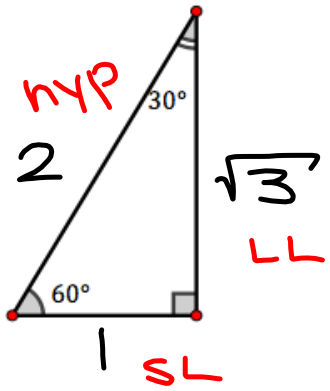
The relationship between the lengths of the sides of a 30 – 60 – 90 triangle

$\frac{LL}{SL} = \frac{\sqrt{3}}{1}$

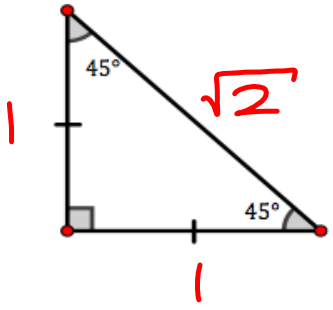
$\frac{LL}{Hyp} = \frac{\sqrt{3}}{2}$

$\frac{SL}{Hyp} = \frac{1}{2}$



The relationship between the lengths of the sides of a 45 – 45 – 90 triangle

$\frac{Hyp}{Leg} = \frac{\sqrt{2}}{1}$



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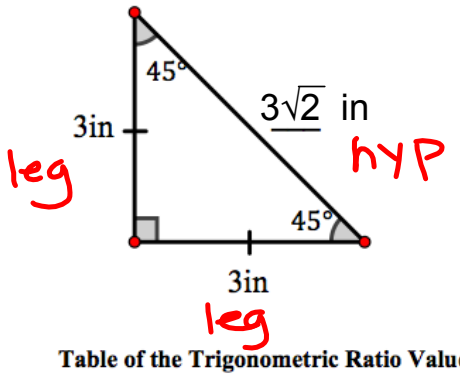
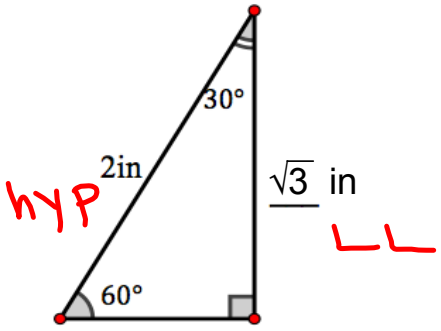
Fill in the blank measurements in the diagrams below. Then use the two special right triangles to fill out the table below. Write all your results in exact form.



leg hyp leg



hyp LL SL

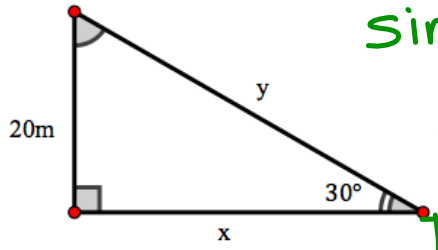
Table of the Trigonometric Ratio Values of Special Angles

(erase to show)

Angle Measure: θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{1}=1$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1}=\sqrt{3}$

This is an important table. It tells us the ratios between the sides of these special right triangles. That will allow us to easily find side lengths without a calculator. If a triangle has these angles we can always use the trig ratios we've identified to solve for any side, provided we are given at least one other side

Example 1 30-60-90



$\sin 30=\frac{20}{y}$

$\frac{1}{2}=\frac{20}{y}$

$y=40m$

$\tan 30=\frac{20}{x}$

$\frac{1}{\sqrt{3}}=\frac{20}{x}$

$x=20\sqrt{3}\ m$

$\frac{LL}{SL}=\frac{x}{20}$

$\frac{\sqrt{3}}{1}=\frac{x}{20}$

$x=20\sqrt{3}\ m$

Trig Table

$\sin 30 = \frac{8}{x}$

$\frac{1}{2} = \frac{8}{x}$

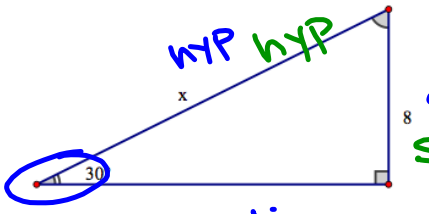
Ratios

$\frac{hyp}{SL} = \frac{x}{8}$

$\frac{2}{1} = \frac{x}{8}$

x = 16 units

Example 2 (30-60-90)



Trig Table

$\sin 60 = \frac{15\sqrt{3}}{x}$

$\frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{x}$

Ratios

$\frac{LL}{hyp} = \frac{15\sqrt{3}}{x}$

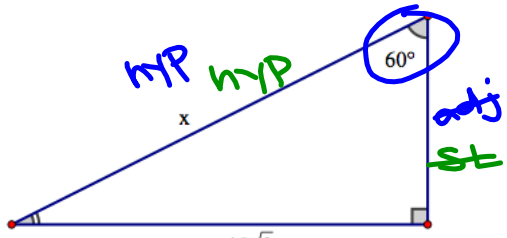
$\frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{x}$

$x \cdot \sqrt{3} = 30\sqrt{3}$

$\sqrt{3} \quad \sqrt{3}$

x = 30 units

Example 3



Trig Table

$\cos 45° = \frac{5\sqrt{2}}{x}$

$\frac{1}{\sqrt{2}} = \frac{5\sqrt{2}}{x}$

Ratios

$\frac{hyp}{leg} = \frac{x}{5\sqrt{2}}$

$\frac{\sqrt{2}}{1} = \frac{x}{5\sqrt{2}}$

$x = 5 \cdot \sqrt{2} \cdot \sqrt{2}$

$x = 5 \cdot \sqrt{4}$

$x = 5 \cdot 2$

x = 10 units

Example 4