Module 4a: Proving the Pythagorean Thm

Math Practice(s):

-Reason abstractly & quantitatively

-Look for & express regularity in repeated reasoning.

Learning Target(s):

-Know the Pythagorean Theorem and its converse.

Homework:

HW: Notes 4b page 1

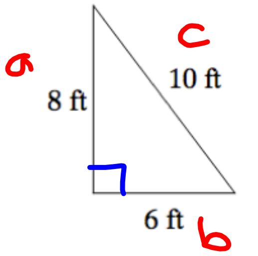
By this time, you should know the relationship between the lengths of the two legs *a* and *b* in a right triangle with its hypotenuse *c*.

The Pythagorean Theorem

In a right triangle with legs of length *a* and *b* and hypotenuse of length *c*,

$a^2 + b^2 = c^2$. #THM

What you may not know is that the Pythagorean Theorem also works in reverse…



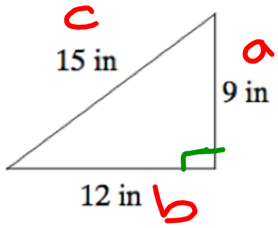
$a^2+b^2=c^2$

$8^2+6^2=10^2$

$64+36=100$

$100=100$ ✓

right △



$9^2+12^2=15^2$

$81+144=225$

$225=225$ ✓

right △

✓: 8, 9, 10
a b c

$8^2+9^2=10^2$

$64+81=100$

$145 \neq 100$

✗

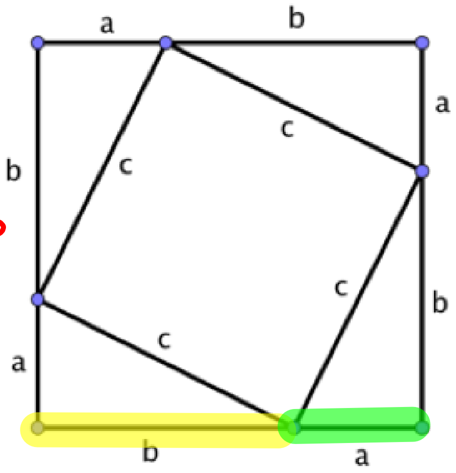
not a right △

The Converse of the Pythagorean Theorem

If the sum of the square of the legs of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.
. #THM

Oct 17-7:36 AM

Let's explore one way to verify that the Pythagorean Theorem is true.



a. What do you notice about the area of the large square?

$A = (a+b)^2$

$\boxed{A = (a+b)(a+b)}$

a+b

a+b

b. What do you notice about the area of the smaller inner square?

$\boxed{A = c^2}$

$A = c \cdot c$

c

c

c. What can we say about the area of the 4 right triangles formed between the large outer square and smaller inner square?

a b c

b

$A = \frac{1}{2}ab$

$\frac{1}{2}bh$

$4\triangle s$

$A = 4\left(\frac{1}{2}ab\right)$

$4 \cdot \frac{1}{2} \cdot a \cdot b$

$\boxed{A = 2ab}$

d. What kind of relationship is there between the area of the large square, smaller square, and four right triangles?

$A_{small\ square} + A_{4\ rt\ \triangle s} = A_{larger\ square}$

$c^2 + 2ab = (a+b)(a+b)$

F L I O

$a^2 + \underline{ab + ab} + b^2$

$c^2 + 2\cancel{ab} = a^2 + 2\cancel{ab} + b^2$

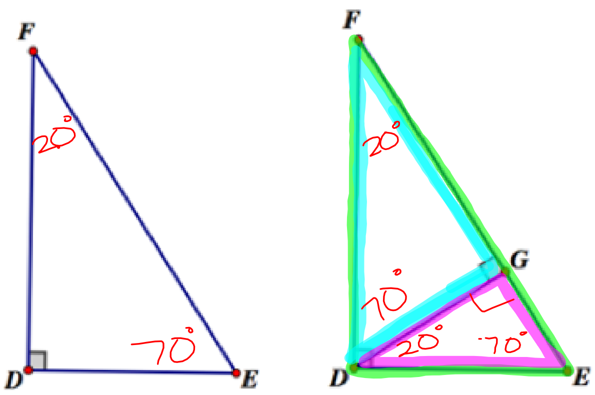
$-2ab \qquad -2ab$

$\boxed{c^2 = a^2 + b^2}$

Now, we will use the similarity of right triangles to prove the Pythagorean Theorem.

Right triangle ΔDEF is shown below. A copy of ΔDEF is also shown. However, an additional segment is drawn in this figure:

- A perpendicular line segment is drawn from D to the hypotenuse $\overline{FE}$.
- The point where the perpendicular line segment meets the hypotenuse is labeled point G.



4. The figure on the right contains 3 triangles.

Name all 3:

- The big (original) triangle:

ΔFED

- The medium triangle:

ΔFGD

- The small triangle:

ΔDEG

a. If $m\angle GFD = 20°$, determine the measures of the following angles and write the angle measures in the diagram above.

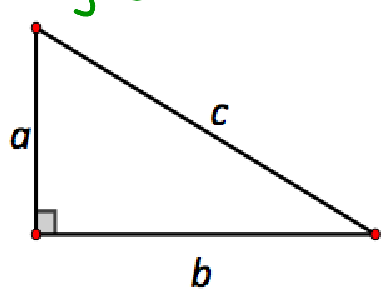
$m\angle FED =$ 70° $m\angle FDG =$ 70° $m\angle GDE =$ 20°

b. Explain why the above angle measures tell you that the three right triangles (the big, medium, and small triangles) are all similar to each other.

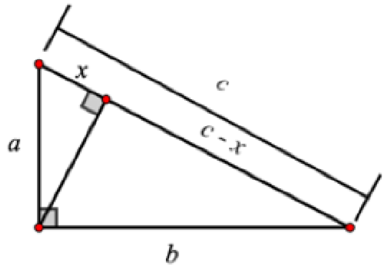
• They all have the same ∠ measures.

* • They have ≅ corresponding ∠s.

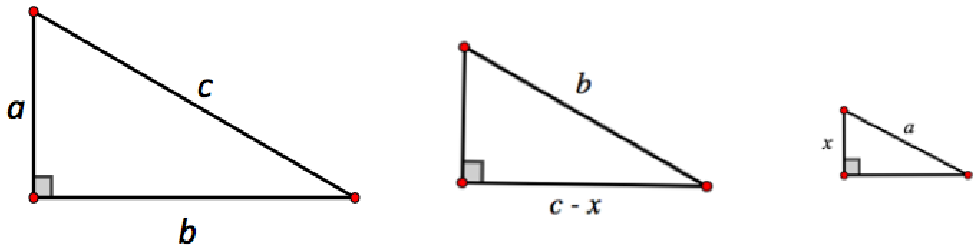
5. Let's simplify this picture a bit, this time just focusing on the lengths of each side. We start with a right triangle with legs of length a and b and hypotenuse of length c.



If we once again drop a perpendicular line segment from the right angle to the hypotenuse, it will cut the hypotenuse into two line segments, the smaller segment with length x and the larger segment with length $c - x$.



As we saw before, this will yield 3 similar triangles: a big one, a medium one, and a small one. They have all been drawn (possibly rotated) so that the right angle is on the bottom-left.



a. Use the fact that the small and big triangles are similar to complete the following proportion:

$$\frac{x}{a} = \frac{a}{c}$$

b. Solve the proportion that you just wrote (solve for x).

$$\frac{\cancel{c}x}{\cancel{c}} = \frac{a^2}{c}$$

$$x = \frac{a^2}{c}$$

c. Use the fact that the medium and big triangles are similar to complete the following proportion:

$$\frac{c-x}{b} = \frac{b}{c}$$

d. Solve the proportion that you just wrote (solve for x).

$$c(c-x) = b \cdot b$$

$$c^2 - cx = b^2$$

$$-c^2 \qquad -c^2$$

$$\frac{\cancel{-c}x}{\cancel{-c}} = \frac{b^2-c^2}{-c}$$

$$x = \frac{b^2-c^2}{-c}$$

e. Use your answers to questions b and d (above) to set up an equation. Then, rearrange the equation to conclude that $a^2 + b^2 = c^2$.

$$\cancel{-c} \cdot \left(\frac{b^2-c^2}{\cancel{-c}}\right) = \left(\frac{a^2}{\cancel{c}}\right) \cdot -\cancel{c}$$

$$\cancel{b^2} - c^2 = -a^2$$

$$\cancel{-b^2} \qquad -b^2$$

$$\frac{-c^2}{-1} = \frac{-a^2}{-1} \frac{-b^2}{-1}$$

$$c^2 = a^2 + b^2$$