Module 3c: The Sine & Cosine Ratios

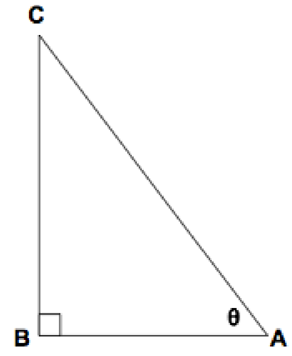
Math Practice(s):
-Reason abstractly & quantitatively
-Construct viable arguments & critique the reasoning of others

Learning Target(s):
-Use trig ratios to determine the lengths of missing sides of a right triangle.

Homework:
HW #12: 3c #1-10

Warm-up

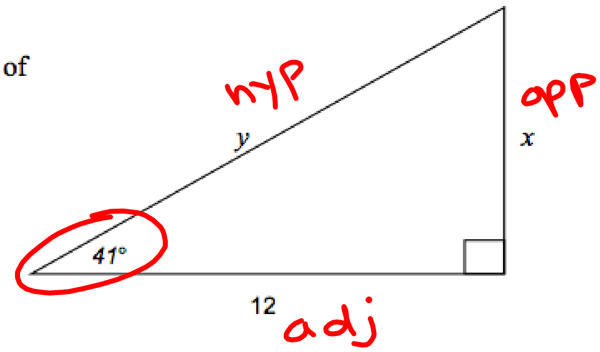
Consider the right triangle ABC. Decide if the following statements are true or false. Explain your answers.



A. If $\tan\theta = \frac{5}{3}$, then $BC = 5$ and $AB = 3$.

yes, it is true because $\overline{BC}$ is opp of θ & tan ratio is $\frac{opp}{adj}$.

2. In the diagram, x represents the length of one of the legs of the right triangle and y represents the length of the hypotenuse.



B. Determine the length of the leg whose measure is unknown. Show how you determined your answer (using tangent ratio). Round your answer to the nearest thousandths.

$$12(\tan 41^\circ) = \left(\frac{x}{\cancel{12}}\right)\cancel{12}$$

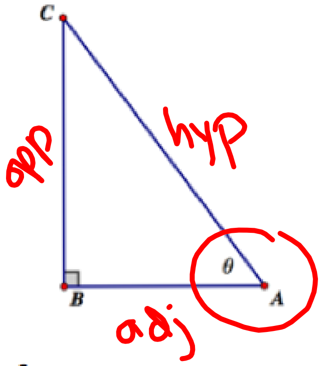
$x \approx 10.431$ units

C. Determine the length of the hypotenuse. Show how you determined your answer (using the tangent ratio). Round your answer to the nearest thousandths.

can't find the length of the hypotenuse by using tangent.

erase to show

The Sine Ratio



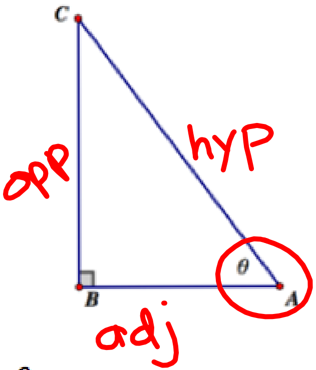
- θ represents the measure of angle A.
  - θ could represent any degree measure (e.g., $\theta = 65°$).
- The expression " $\sin\theta$ " is pronounced, "sine of theta".
- To determine the number that " $\sin\theta$ " represents, we have to set up the following ratio:

$$\sin\theta = \frac{\textit{length of the side OPPOSITE from } \theta}{\textit{length of the HYPOTENUSE}}$$

➔ This is often abbreviated in a simpler form as

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

The Cosine Ratio



- θ represents the measure of angle A.
  - θ could represent any degree measure (e.g., $\theta = 31.5°$).
- The expression " $\cos\theta$ " is pronounced, "cosine of theta".
- To determine the number that " $\cos\theta$ " represents, we have to set up the following ratio:

$$\cos\theta = \frac{\textit{length of the side ADJACENT to } \theta}{\textit{length of the HYPOTENUSE}}$$

➔ This is often abbreviated in a simpler form as

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

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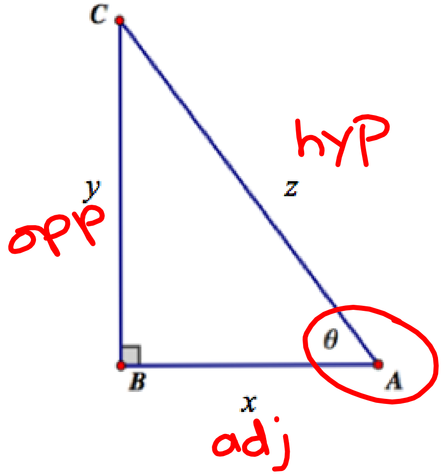
Example 1: In the diagram shown, x, y and z represent the lengths of one side of the right triangle, and θ represents the measure of $\angle A$



A. Set-up an equation using the sine ratio.

$$\sin\theta = \frac{y}{z}$$

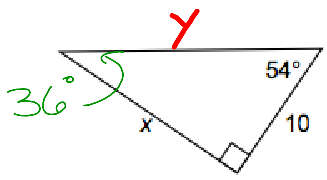
B. Set-up an equation using the cosine ratio.

$$\cos\theta = \frac{x}{z}$$

C. Set-up an equation using the tangent ratio.

$$\tan\theta = \frac{y}{x}$$

Example 2: In the diagram shown, x represent the lengths of one leg of the right triangle
y represents length of hypotenuse



A. Set-up 2 equations using the sine ratio.

$$\sin 54° = \frac{x}{y}$$

$$\sin 36° = \frac{10}{y}$$

B. Set-up 2 equations using the cosine ratio.

$$\cos 54° = \frac{10}{y}$$

$$\cos 36° = \frac{x}{y}$$

C. Set-up 2 equations using the tangent ratio.

$$\tan 54° = \frac{x}{10}$$

$$\tan 36° = \frac{10}{x}$$

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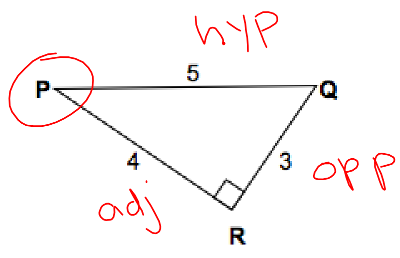
Example 3: For each right triangle below, determine the value of sin P, sin Q, cos P, and cos Q.

a. $\sin P = \frac{3}{5}$

$\sin Q = \frac{4}{5}$

$\cos P = \frac{4}{5}$

$\cos Q = \frac{3}{5}$



b. $\sin P = \frac{2\sqrt{10}}{7}$

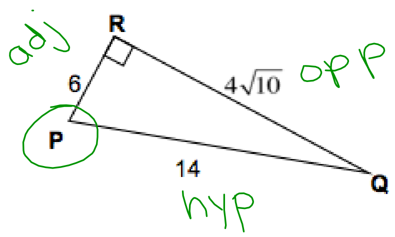
$\frac{4\sqrt{10}}{14}$

$\sin Q = \frac{3}{7}$

$\frac{6}{14}$

$\cos P = \frac{3}{7}$

$\cos Q = \frac{2\sqrt{10}}{7}$



$\frac{^{1}\cancel{4}\sqrt{11}}{6\,\cancel{24}}$

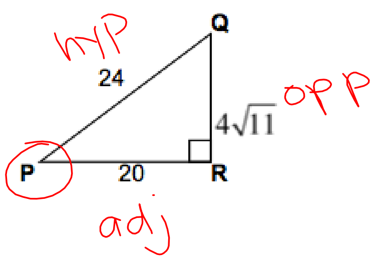
c. $\sin P = \frac{\sqrt{11}}{6}$

$\frac{20}{24}$

$\sin Q = \frac{5}{6}$

$\cos P = \frac{5}{6}$

$\cos Q = \frac{\sqrt{11}}{6}$



erase to show

Reflection

Referring to the triangles and your answers in Example 3 (on the previous page), work with a partner to discuss and answer the following questions.

a. In each of the triangles on the previous page, since *P* and *Q* are acute angles in a right triangle, their measures add up to $90°$. Thus, they are called complementary angles.

b. For each triangle on the previous page,

- compare the value of sin *P* to the value of cos *Q* and,
- compare the value of cos *P* to the value of sin *Q*.

What do you notice? Write a complete sentence to summarize your thoughts (your sentence should include the following words: sine, cosine and complementary angles).

The sin P & cos Q are the same, because the acute ∠s are complementary.

c. Translate your sentence above (your answer for question b) into equations. Fill in the blanks to make two true statements.

If *P* and *Q* are complementary angles, then

sin P = cos Q *and* sin Q = cos P

Trigonometry is the study of measurements using triangles.

- tri ➔ means "3"
- gon ➔ comes from the Greek word "gōnia" which means "angle"
- metric ➔ comes from the Greek word "metron" which means "measurement"

Therefore, the literal translation of TRIGONOMETRY is "3 angle measurement".

The sine, cosine and tangent ratios are the three main trigonometric ratios and are very useful for solving a wide variety of real-world problems.

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Practice

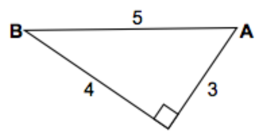
2. Determine if the following trigonometric ratios were used correctly in each equation.

If it is correct, circle the equation. If it is NOT correct, rewrite the equation so it is correct.

A. $\sin A = \frac{4}{5}$

B. $\cos A = \frac{5}{3}$

$\cos A = \frac{3}{5}$



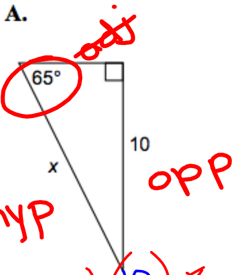
C. $\sin B = \frac{3}{4}$

D. $\cos B = \frac{4}{5}$

$\sin B = \frac{3}{5}$

3. Determine the value of x in each of the triangles below. Show how you determined your answer using a trigonometric ratio. Round your answer to the thousandths place.

A.



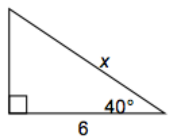
$\sin 65° = \left(\frac{10}{x}\right)$

$\frac{x \cdot \cancel{\sin 65°}}{\cancel{\sin 65°}} = \frac{10}{\sin 65°}$

$\boxed{x \approx 11.034 \text{ units}}$

11.0337

B.

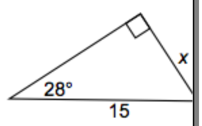


$x \cdot (\cos 40°) = \left(\frac{6}{x}\right) \cdot x$

$\frac{x \cdot \cancel{\cos 40°}}{\cancel{\cos 40°}} = \frac{6}{\cos 40°}$

$\boxed{x \approx 7.832 \text{ units}}$

C.

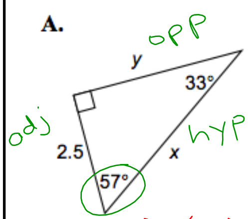


$15 \cdot (\sin 28°) = \left(\frac{x}{\cancel{15}}\right) \cancel{15}$

$\boxed{x \approx 7.042 \text{ units}}$

4. Determine the value of x and y in each of the triangles below. Show how you determined your answers using trigonometric ratios. Round your answers to the thousandths place.

A.



$2.5 \cdot (\tan 57°) = \left(\frac{y}{\cancel{2.5}}\right) \cancel{2.5}$

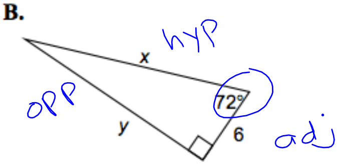
$\boxed{y \approx 3.850 \text{ units}}$

$x \cdot (\cos 57°) = \left(\frac{2.5}{\cancel{x}}\right) \cancel{x}$

$\frac{x \cdot \cancel{\cos 57°}}{\cancel{\cos 57°}} = \frac{2.5}{\cos 57°}$

$\boxed{x \approx 4.590 \text{ units}}$

B.



$6 \cdot (\tan 72°) = \left(\frac{y}{\cancel{6}}\right) \cancel{6}$

$\boxed{y \approx 18.466 \text{ units}}$

$x \cdot (\cos 72°) = \left(\frac{6}{\cancel{x}}\right) \cancel{x}$

$\frac{x \cdot \cancel{\cos 72°}}{\cancel{\cos 72°}} = \frac{6}{\cos 72°}$

$\boxed{x \approx 19.416 \text{ units}}$