# Module 3b: The Tangent Ratio

**Math Practice(s):**

-Reason abstractly & quantitatively

-Construct viable arguments & critique the reasoning of others

**Learning Target(s):**

-Use trig ratios to determine the lengths of missing sides of a right triangle.

**Homework:**

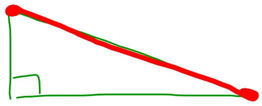
HW #11: 3a #1-5

## Warm-up

Work with a partner to discuss and then answer the following questions.

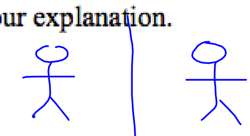
1. In your own words, briefly explain what the word **hypotenuse** means. Draw and label a diagram to provide a visual representation your explanation.

The long side of a right △, opposite of the right ∠.



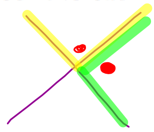
2. In your own words, briefly explain what it means if the location of two objects are **opposite** from each other. Draw a diagram to provide a visual representation your explanation.

reflection
+5 vs −5 on a # line



3. In your own words, briefly explain what it means if two angles are **adjacent** to each other. Draw a diagram to provide a visual representation your explanation.

If 2 ∠s share vertex & side



4. A lot of the ideas that we learn in Geometry were studied and formalized by Greek mathematicians over 2000 years ago. Thus, sometimes symbols from the Greek alphabet are used as variables (instead of letters from the English alphabet).

On the left are a few symbols (letters) from the Greek alphabet, and on the right are letters from the English alphabet. Draw a line to match each Greek letter with its corresponding English letter. (Even if you don't know for sure, use your intuition to make an educated guess.)

| Greek Alphabet Symbol | English Equivalent |
|---|---|
| $\pi$ "pi" | a |
| $\varepsilon$ "epsilon" | b |
| $\alpha$ "alpha" | e |
| $\theta$ "theta" | p |
| $\beta$ "beta" | th |

Sep 12-11:01 AM

erase to show

***Example 1***

Consider the right triangle $\Delta ABC$. The measure of $\angle A$ is represented by the Greek symbol $\theta$ ("theta"). Fill in the blanks with the name of the appropriate side (using line segment notation).

(hyp)

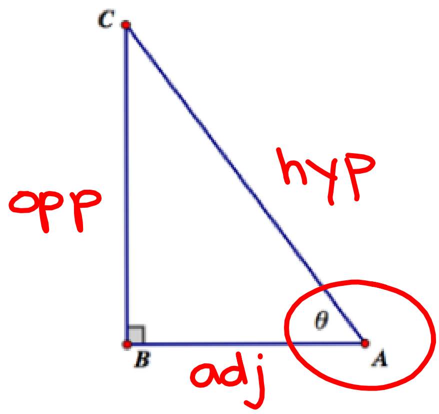
- The hypotenuse of the triangle is $\overline{AC}$.

(opp)

- The leg *opposite* to $\theta$ is $\overline{BC}$.

(adj)

- The leg *adjacent* to $\theta$ is $\overline{AB}$.




***Example 2***

Consider the right triangle $\Delta ABC$ below. The measure of $\angle A$ is represented by the Greek symbol $\alpha$ ("alpha") and the measure of $\angle B$ is represented by the Greek symbol $\beta$ ("beta"). Fill in the blanks with the name of the appropriate side (using line segment notation).

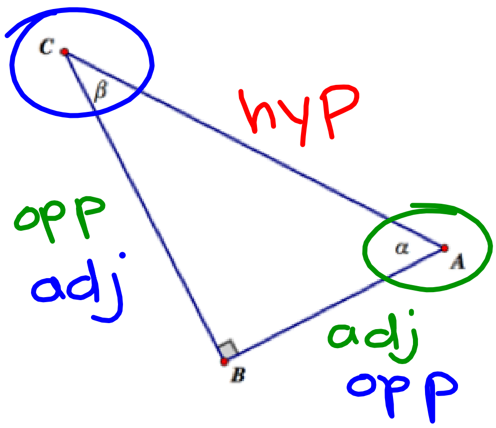
a. The hypotenuse of the triangle is $\overline{AC}$.

b. The leg opposite to $\alpha$ is $\overline{BC}$.

c. The leg adjacent to $\alpha$ is $\overline{AB}$.

d. The leg opposite to $\beta$ is $\overline{AB}$.

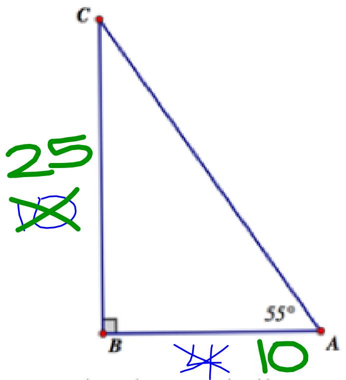
e. The leg adjacent to $\beta$ is $\overline{BC}$.

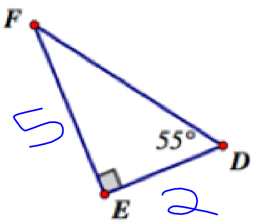



- Since $\alpha$ and $\beta$ represent the measures of the two acute angles in a right triangle, they have a sum of $90°$, and therefore, we can say that $\angle A$ and $\angle C$ are complementary.

***Example 3***

Consider the below two right triangles: $\Delta ABC$ and $\Delta DEF$.




a. The two triangles are similar. Explain why this must be true.

The $\Delta$s both have a rt $\angle$ & a $55°$ $\angle$ in same spot, so the last $\angle$ must be $\cong$. By definition, similar $\Delta$s have $\cong$, corresponding $\angle$s.

b. Both triangles have an angle with measure $55^\circ$.

erase to show

i. The leg opposite to the $55^\circ$ in $\Delta ABC$ is $\overline{BC}$ .

ii. The leg adjacent to the $55^\circ$ in $\Delta ABC$ is $\overline{AB}$ .

iii. The leg opposite to the $55^\circ$ in $\Delta DEF$ is $\overline{EF}$ .

iv. The leg adjacent to the $55^\circ$ in $\Delta DEF$ is $\overline{ED}$ .

c. Use similarity to explain the relationship between the ratio $\frac{BC}{AB}$ of opposite to adjacent legs in $\Delta ABC$ to the ratio $\frac{EF}{DE}$ of opposite to adjacent legs in $\Delta DEF$.

$\frac{BC}{AB} = \frac{5}{2}$ $\quad$ $\frac{EF}{DE} = \frac{5}{2}$ $\quad$ The ratios are equal

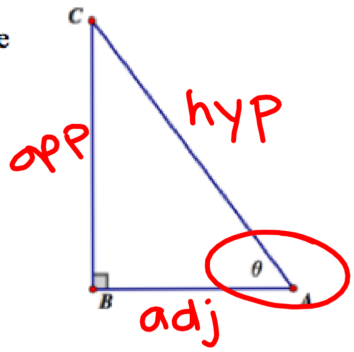
d. If there were another right triangle $\Delta GHI$ that had an angle of $55^\circ$, what would be true of the ratio of its opposite leg to its adjacent leg?

The ratio would be equal to both $\frac{BC}{AB}$ & $\frac{EF}{DE}$.

erase to show

## The Tangent Ratio

In the diagram, the Greek symbol $\theta$ represents the measure of one of the acute angles of the right triangle.




- $\theta$ represents the measure of angle $A$.
  - $\theta$ could represent any degree measure (e.g., $\theta = 30°$ or $\theta = 45°$).
- The expression " $\tan\theta$ " is pronounced, "tangent of theta".
- The expression " $\tan\theta$ " represents a number.
- To determine the number that " $\tan\theta$ " represents, we have to set up a ratio that compares the lengths of the two legs of the right triangle:

$$\tan\theta = \frac{\textit{length of the side } \textbf{\textit{OPPOSITE}} \textit{ from } \theta}{\textit{length of the side } \textbf{\textit{ADJACENT}} \textit{ to } \theta}$$

➔ This is often abbreviated in a simpler form as

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

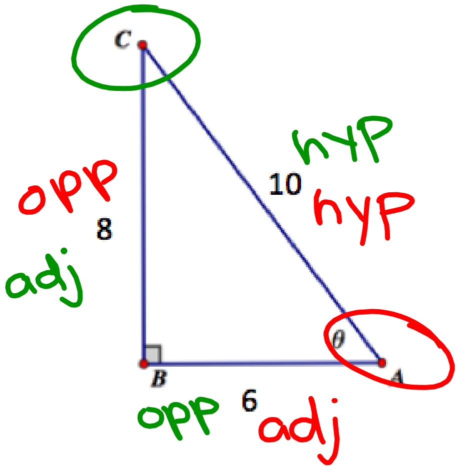
***Example 4***

- $\tan\theta = \frac{8}{6}$

$$\boxed{\tan\theta = \frac{4}{3}}$$

- $\tan C = \frac{6}{8}$

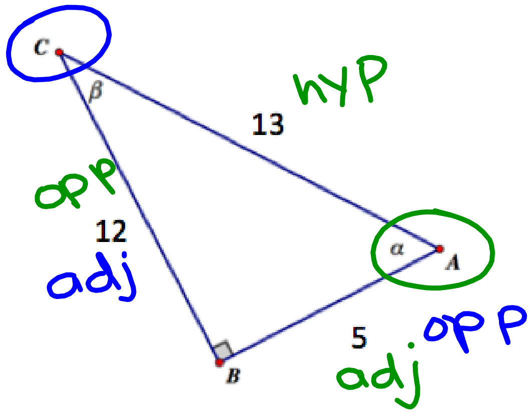
$$\boxed{\tan C = \frac{3}{4}}$$



***Example 5***

- $\tan\alpha = \frac{12}{5}$

- $\tan\beta = \frac{5}{12}$




$\frac{1}{3} \div \frac{7}{4} \rightarrow \frac{1}{3} \cdot \frac{4}{7} = \frac{4}{21}$

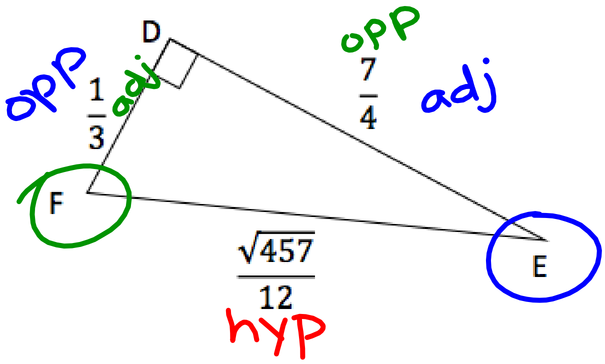
***Example 6***

- $\tan E = \frac{\frac{1}{3}}{\frac{7}{4}}$

$\boxed{\tan E = \frac{4}{21}}$

- $\tan F = \frac{\frac{7}{4}}{\frac{1}{3}}$

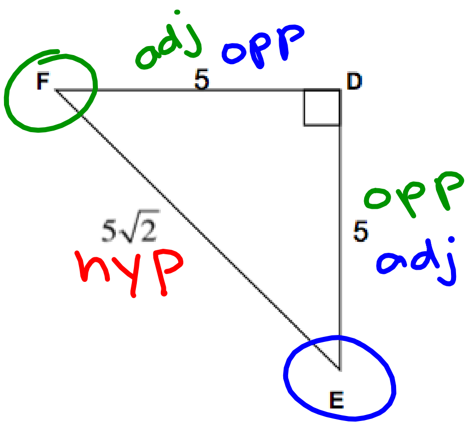
$\boxed{\tan F = \frac{21}{4}}$




***Example 7***

- $\tan E = \frac{5}{5}$ $\boxed{\tan E = 1}$

- $\tan F = \frac{5}{5}$ $\boxed{\tan F = 1}$




Sep 12-11:02 AM

# Homework

**Reflection:** Work with a partner to discuss and answer the following questions.

**a.** In examples 4 – 7, you determined the tangent ratio for the two acute angles in each triangle. Analyze each pair of tangent ratios and make a conclusion about the relationship between the tangent ratios of the acute angles of a right triangle.

The tangent ratios are the reciprocals of each other.

**b.** In a right triangle, $\alpha$ and $\beta$ represent the measures of the two acute angles. If $\tan \alpha = \frac{4}{7}$, what is the value of $\tan \beta$?

$\tan B = \frac{7}{4}$

**c.** In a right triangle, $\alpha$ and $\beta$ represent the measures of the two acute angles.

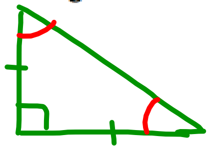
**i.** If $\tan \alpha = 1$, what is $\tan \beta$ ? $\tan B = 1$

**ii.** If $\tan \alpha = 1$, what could you conclude about the two legs (the opposite and adjacent legs) of your triangle?

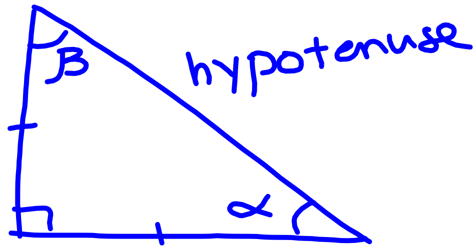
The legs are $\cong$ (the same length)

**iii.** If $\tan \alpha = 1$, what must be the measure of both of the acute angles of this right triangle?

45°



**iv.** Draw and label a right triangle that represents your answers to questions i – iii.

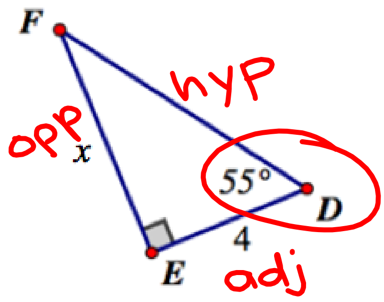



Sep 12-11:03 AM

**Homework: Write the equations for the rest of the problems (see Ex 8).**

***Example 8***

In the diagram, $x$ represents the length of $\overline{FE}$.




- Set-up an equation using the tangent ratio. Then, solve your equation to determine the value of $x$.

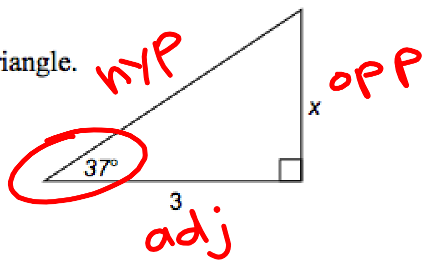
  Round your answer to the nearest thousandth; however, round **ONLY after** your final computation.

$$4(\tan 55^\circ) = \left(\frac{x}{\cancel{4}}\right)\cdot \cancel{4}$$

$$\boxed{x \approx 5.713}$$

***Example 9***

In the diagram, $x$ represents the length of one of the legs of the right triangle.




- Set-up an equation using the tangent ratio. Then, solve your equation to determine the value of $x$.

  Round your answer to the nearest thousandth; however, round **ONLY after your final computation**.

$$3\cdot(\tan 37^\circ) = \left(\frac{x}{\cancel{3}}\right)\cancel{3}$$

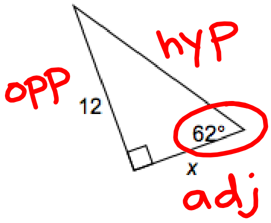
3* tan (37) $\boxed{\text{enter}}$

$$\boxed{x \approx 2.261}$$

**Practice**: In each diagram below, *x* represents the length of one of the legs of the right triangle.

- Set-up an equation using the tangent ratio. Then, solve your equation to determine the value of *x*.
- Round your answer to the nearest thousandth.

**5.**

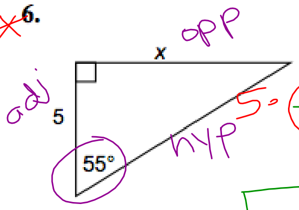



$\cancel{x} \cdot (\tan 62°) = \left(\frac{12}{\cancel{x}}\right) \cdot \cancel{x}$

$\frac{x \cdot \cancel{\tan 62°}}{\cancel{\tan 62°}} = \frac{12}{\tan 62°}$

$x \approx 6.381$

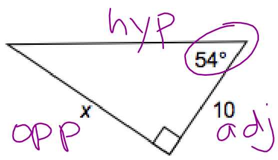
**6.**




$5 \cdot (\tan 55°) = \left(\frac{x}{\cancel{5}}\right)\cancel{5}$

$\boxed{x \approx 7.141}$

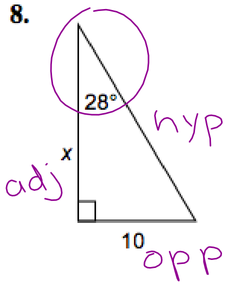
**7.**




$10(\tan 54°) = \left(\frac{x}{\cancel{10}}\right)\cancel{10}$

$\boxed{x \approx 13.764}$

**8.**

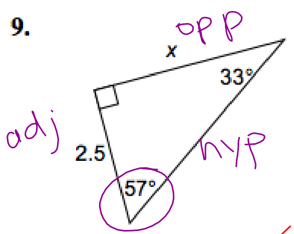



$\cancel{x}(\tan 28°) = \left(\frac{10}{\cancel{x}}\right)\cancel{x}$

$\frac{x \cdot \cancel{\tan 28}}{\cancel{\tan 28}} = \frac{10}{\tan 28}$

$\boxed{x \approx 18.807}$

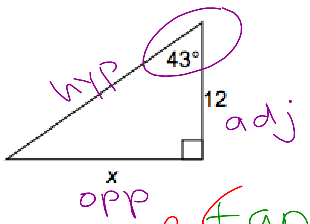
**9.**




$2.5(\tan 57°) = \left(\frac{x}{\cancel{2.5}}\right)\cancel{2.5}$

$\boxed{x \approx 3.850}$

**10.**




$12(\tan 43°) = \left(\frac{x}{\cancel{12}}\right)\cancel{12}$

$\boxed{x \approx 11.190}$