Quadratics 5a - The Quadratic Formula

Standards N-CN.7, A-REI.4a, A-REI.4b, F-IF.7a

Math Practices: Attend to Precision

GLO: **#3** - Complex Thinker

Learning Targets:
How do you use the Quadratic Formula to solve?
How do you graph a quadratic in Standard Form?

Oct 14-11:29 PM

What if we can't solve by factoring or square rooting?

One more way to solve...

Quadratic Formula

the solutions of $ax^2+bx+c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This means that we can determine the solutions for any quadratic equation by setting it equal to zero and substituting the values of ***a***, ***b***, & ***c*** into the formula.

Remember: You can ONLY use this formula to **solve** a quadratic equation in **standard form**!

Example 1: Solve $2x^2 + 3x - 9 = 0$

First, we must identify the values of a, b, & c:

$\boldsymbol{a = 2} \qquad \boldsymbol{b = 3} \qquad \boldsymbol{c = -9}$

Now, substituting these values into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-9)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 - (-72)}}{4}$$

$$x = \frac{-3 \pm \sqrt{81}}{4}$$

$$x = \frac{-3 \pm 9}{4}$$

$$x = \frac{-3+9}{4} \quad \text{or} \quad x = \frac{-3-9}{4}$$

$$x = \frac{6}{4} \quad \text{or} \quad x = \frac{-12}{4}$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -3$$

Oct 21-10:42 PM

Example 2:

a) $-8x^2 - 5x = -\cancel{x^2} - \cancel{1}$

$+x^2 \quad +1 \quad +x^2 \quad +1$

$-7x^2 - 5x + 1 = 0 \qquad a=-7,\ b=-5,\ c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2-4(-7)(1)}}{2(-7)}$$

$$x = \frac{5 \pm \sqrt{25++28}}{-14}$$

$$x = \frac{5 \pm \sqrt{53}}{-14}$$

$$\boxed{x = \frac{5 \pm \sqrt{53}}{-14}}$$

OR

$$\boxed{x = \frac{5+\sqrt{53}}{-14} \qquad x = \frac{5-\sqrt{53}}{-14}}$$

2 real solutions

b) $2x^2 - 5x + 7 = \cancel{x^2} - \cancel{3}x + \cancel{6}$

$-x^2 \quad +3x \quad -6 \quad -x^2 \quad +3x \quad -6$

$x^2 - 2x + 1 = 0 \qquad a=1,\ b=-2,\ c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4-4}}{2}$$

$$x = \frac{2 \pm \sqrt{0}}{2}$$

$$x = \frac{2}{2}$$

$x = 1$

1 real solution

c) $3x^2 - 3x + 5 = 0 \qquad a=3,\ b=-3,\ c=5$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(3)(5)}}{2(3)}$$

$$x = \frac{3 \pm \sqrt{9-60}}{6}$$

$$x = \frac{3 \pm \sqrt{-51}}{6}$$

$\sqrt{-1 \cdot 51}$

$\sqrt{-1} \cdot \sqrt{51}$

$i \cdot \sqrt{51}$

$$x = \frac{3 \pm \sqrt{51}\, i}{6}$$

$$x = \frac{3}{6} \pm \frac{\sqrt{51}}{6} i$$

$$\boxed{x = \frac{1}{2} \pm \frac{\sqrt{51}}{6} i}$$

OR

$$\boxed{x = \frac{1}{2} + \frac{\sqrt{51}}{6} i \qquad x = \frac{1}{2} - \frac{\sqrt{51}}{6} i}$$

No real solutions

2 Imaginary solutions

A consequence of the quadratic formula is that if $f(x) = ax^2 + bx + c$ has an x-intercept, then the x-intercept(s) must be located at points on the x-axis with x-coordinate equal to $\frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

Notice: This formula states that one x-intercept is located at $\frac{-b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$ and the other is located at $\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$ so they are equal distances from $\frac{-b}{2a}$.

Important Consequence:

This implies that $\frac{-b}{2a}$ must be the x-coordinate of the vertex!

So the vertex is at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Graphing from Standard Form:

To find the x-intercepts:

- Set the equation equal to zero (y=0). Use the method of your choice to solve for x.
- If the equation is in standard form ($ax^2 + bx + c$), then you can solve by **factoring** or the **quadratic formula**.
- If the equation is in factored form ($a(x-s)(x-t)$), then you can solve by just using the **zero-product property**.
- If the equation is in basic form ($ax^2 + c$) or vertex form ($a(x - h)^2 + k$), then you can solve by **square rooting.**

The resulting solutions are the x-values of the x-intercepts. You may need to use a scientific calculator. If you do, write your values accurate to one decimal places.

- The y-value is zero.

$$x = \frac{-b}{2a}$$

To find the y-intercept:

- Set $x = 0$ and solve for y.
- The c-value of a quadratic function written in standard form gives you the y-value of the y-intercept.

To find the vertex:

- Look at your calculations from when you used the Quadratic Formula. $\left(\frac{-b}{2a}\right)$

The beginning portion before the ± sign, the tells you the x-value of the vertex.

- To determine the y-value of the vertex, plug in the x-value into the function and evaluate.

Once these key points are determined, all that remains is to find additional points to complete the graph.

Example 3:

Graph $f(x) = 2x^2 - 8x + 3$.

x-int: (set y=0 & solve)

$0 = 2x^2 - 8x + 3$ $a = 2$, $b = -8$, $c = 3$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$

$x = \frac{8 \pm \sqrt{64 - 24}}{4}$

$x = \frac{8 \pm \sqrt{40}}{4}$

$x = \frac{8 + \sqrt{40}}{4} \approx 3.6$

$x = \frac{8 - \sqrt{40}}{4} \approx 0.4$

y-int: (c-value)

$c = 3$

Vertex:

$x: \frac{-b}{2a} \rightarrow \frac{-(-8)}{2(2)} \rightarrow \frac{8}{4} \rightarrow 2$

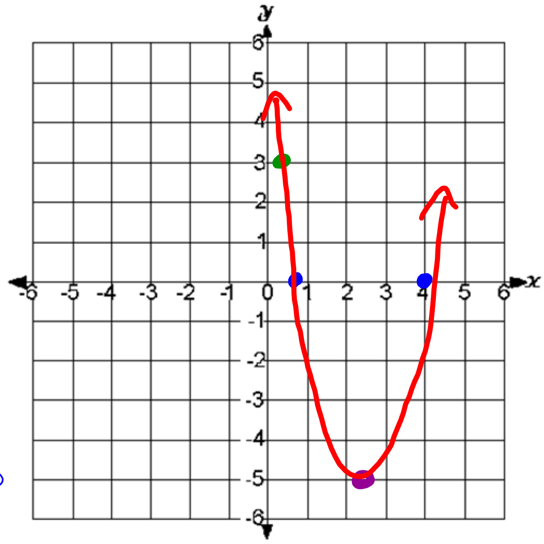
$y: f(2) = 2(2)^2 - 8(2) + 3$

$= 2(4) - 16 + 3$

$= 8 - 16 + 3$

$= -8 + 3$

$f(2) = -5$



x-int: $(3.6, 0)$ & $(0.4, 0)$

y-int: $(0, 3)$

vertex: $(2, -5)$

domain: all real numbers

range: $y \geq -5$

$f(x) \geq -5$

Example 4:

Graph $f(x) = -x^2 + 6x - 3$

x-int:

$0 = -x^2 + 6x - 3$

$a = -1$
$b = 6$
$c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(-1)(3)}}{2(-1)}$$

$$x = \frac{-6 \pm \sqrt{36 + +12}}{-2}$$

$$x = \frac{-6 \pm \sqrt{48}}{-2}$$

$$x = \frac{-6 + \sqrt{48}}{-2} \approx -0.5$$

$$x = \frac{-6 - \sqrt{48}}{-2} \approx 6.5$$

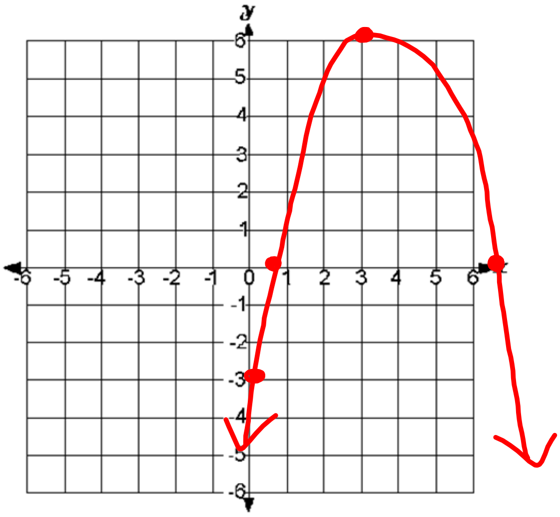
y-int: (c-value)

$c = -3$

vertex:

$$x: \frac{-b}{2a} = \frac{-(6)}{2(-1)} = \frac{-6}{-2} = 3$$

$$y: f(3) = -(3)^2 + 6(3) - 3$$
$$= -(9) + 18 - 3$$
$$= -9 + 18 - 3$$
$$= 9 - 3$$
$$f(3) = 6$$



x-int: $(-0.5, 0)$ & $(6.5, 0)$

y-int: $(0, -3)$

vertex: $(3, 6)$

domain: all real numbers

range: $y \leq 6$
$f(x) \leq 6$