Module 1f: Constructing Right Angles

Math Practice(s):

-Use appropriate tools strategically
-Attend to precision

Learning Target(s):

-Understand that the perpendicular bisector of a segment always exists.
-Be able to construct the perpendicular bisector of a segment to create right angles.

Homework:

HW #6: 1f #1-2

*The arcs shown in these notes are to show you the "work" that I expect to see when you do a construction. For more direction on how to actually construct a right triangle, please see the video posted on our weebly website.

In the previous section, we saw that if we were given a line segment in the plane, we could use the "negative reciprocal" relationship between its slope and the slope of a perpendicular line to construct perpendicular line segments. Of course, to be able to do this, we needed to be *in the coordinate plane* since we needed to be able to compute slope.

In this section, we will see how to *construct* a line perpendicular to a line segment that is not in the coordinate plane. The goal is to use the most basic tools to perform these constructions.

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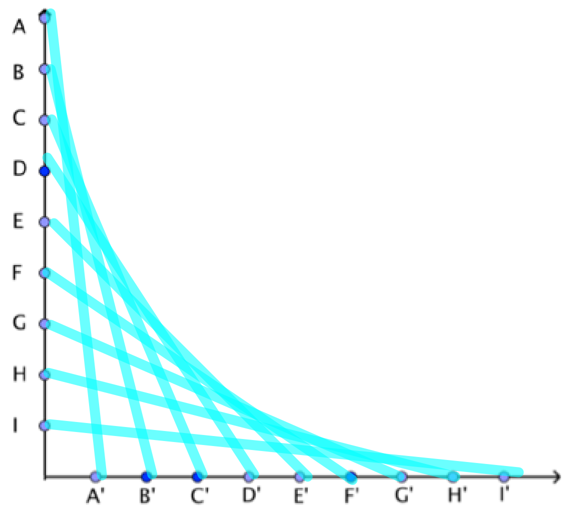
<u>Tool 1: The Straightedge</u> (a ruler with no measurement markings)

We have already used a straightedge to connect points both in and outside of a plane. Between every two distinct points, there exists a unique line segment connecting them, and the straightedge allows us to construct that line segment.

1) Let's practice below by using a straightedge to connect points *A* to *A'*, *B* to *B'*, *C* to *C'*, etc. At the end, you should have 9 line segments on the plane below. "A to A prime"



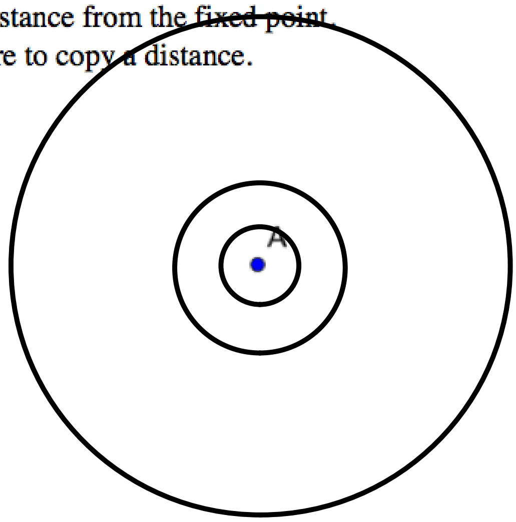
Notice that even though we only used *straight lines*, we used enough of them in a certain pattern that it gave the impression of a *curve*.

<u>Tool 2: The Compass</u>

A compass is any tool that can be used to *copy distances*. The compass in your classroom usually has one fixed part (doesn't move) and a part that swings around (where a pencil can be attached). The compass has some nice properties:

- Every point that your pencil draws is the same distance from the fixed point.
- You can pick up a compass and move it elsewhere to copy a distance.

2) In the space to the right, draw three different *concentric circles* (circles with the same center) with center at *A*.



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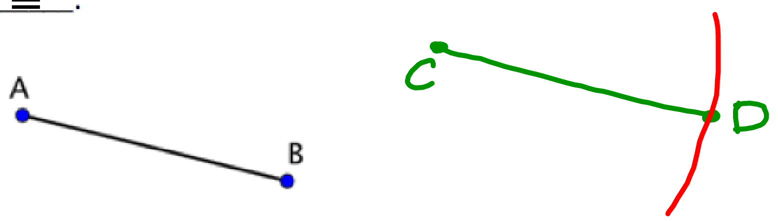
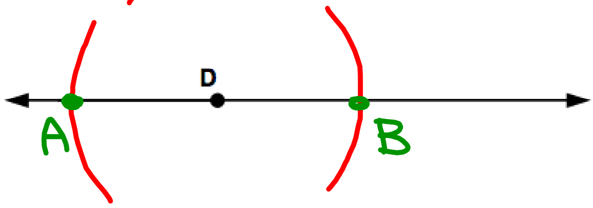
3) Use your compass to create a copy of line segment $\overline{AB}$ elsewhere on the page. Call the new line segment $\overline{CD}$. Notice that $AB = CD$ (their measures are _equal_), so these two lines segments are _$\cong$_.



4) Use your compass to find two points A and B, so that the given point D is the midpoint of $\overline{AB}$.



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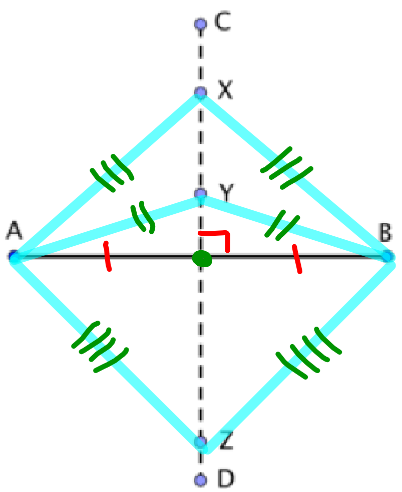
In geometry, a construction is a procedure which uses only simple tools (_straightedge_ and _compass_) that allows us to copy or create lines, angles, or shapes accurately.

In the diagram below, $\overline{CD}$ is the perpendicular bisector for $\overline{AB}$

line, segment, or ray that is perpendicular to a segment at its midpoint.



The perpendicular bisector $\overline{CD}$ contains three other points on it (X, Y, and Z). Use a ruler to measure the distances (in mm) from the endpoints A and B to these three points:

AX = 36mm BX = 36mm

AY = 28mm BY = 28mm

AZ = 39mm BZ = 39mm

*What do you notice about the distance between the endpoints of $\overline{AB}$ (points A and B) and any point on the perpendicular bisector of $\overline{AB}$?

They are the same.

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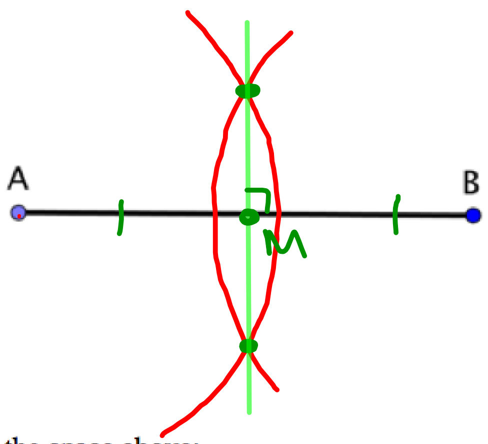
Now let's use your observation that…

any point on a perpendicular bisector of $\overline{AB}$ is ___equidistant___ **from both *A* and *B*.**
(the same distance)

Constructing a Perpendicular Bisector of a Line Segment



Complete the following steps in the space above:

- Place the fixed part of your compass on *A*, and open it to have a distance of more than half the length of *AB*.
- With the fixed part remaining on *A*, place a mark (an arc) from above $\overline{AB}$ to below $\overline{AB}$.
- Using the same distance that you used for step 2, place your compass on point *B* and place a mark (an arc) from above $\overline{AB}$ to below $\overline{AB}$ so that it intersects your previous arc.
- The two arcs you drew intersect at two distinct points (one above $\overline{AB}$ and one below). Connect these two intersection points using a straight edge. This is your perpendicular bisector.

Discuss the following questions with a partner and write down some of the ideas you discussed:

A. Consider the intersection point of the two arcs above $\overline{AB}$. How do you know this point is equidistant to *A* and *B*?
The radius of both circles are the same.

B. Your new perpendicular bisector should intersect $\overline{AB}$ at a point (let's call it *M*) on $\overline{AB}$. Measure the distance *AM* and *BM*. Can you conclude that *M* is the midpoint of $\overline{AB}$? Why or why not.
M is the midpoint of AB, because AM = BM.

C. Since *M* is a point on the perpendicular bisector, how can you use a property of the perpendicular bisector to conclude that *M* is the midpoint of $\overline{AB}$ without actually measuring any distances?
Since every point on the ⊥ bisector of a segment is equidistant from points A & B, M is equidistant from points A & B creating a midpoint.

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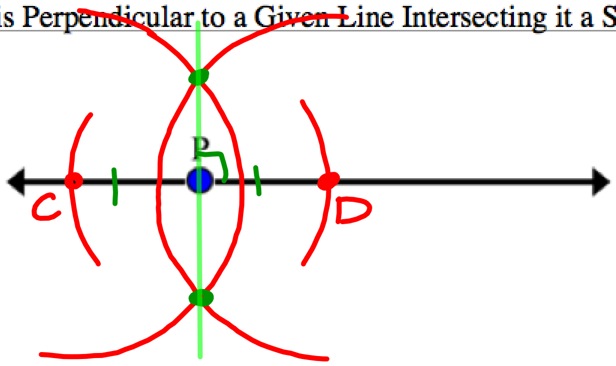
Constructing a Line that is Perpendicular to a Given Line Intersecting it a Specific Point



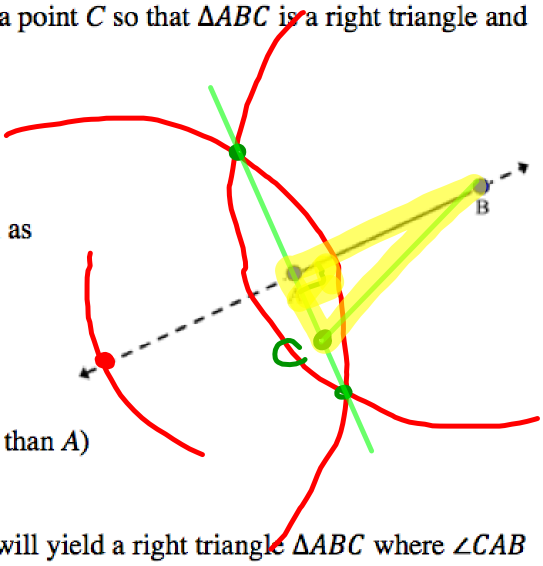
Complete the following steps:

- Place the fixed part of your compass on P and mark off a point that is to one side of P on the line. Label that point C.
- Using that same distance and with the fixed part of your compass still on P, mark off a point on the other side of P that lies on the line. Label that point D.
- Run the usual perpendicular bisector construction on the line segment $\overline{CD}$.

Let's construct a right triangle!

We will start with a line segment $\overline{AB}$. Our goal is to find a point C so that ΔABC is a right triangle and $\angle CAB$ is right.

Complete the following steps:



- Extend the line segment $\overline{AB}$ to the line $\overleftrightarrow{AB}$ (shown as a dashed line).
- Construct a line perpendicular to $\overleftrightarrow{AB}$ that goes through A (using the steps learned previously).
- Choose any point on this perpendicular line (other than A) and label it C.
- Connect C to A and B using a straight edge. This will yield a right triangle ΔABC where $\angle CAB$ is right.

Discuss the following question with a partner and write down some of the ideas you discussed:

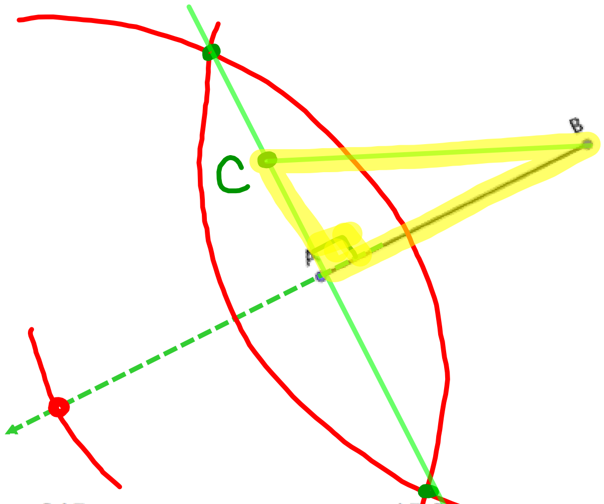
➢ You and your friend both do this construction independently. Will you both construct the exact same triangle. Why or why not?

No, because we will probably choose different places to put C.

Practice

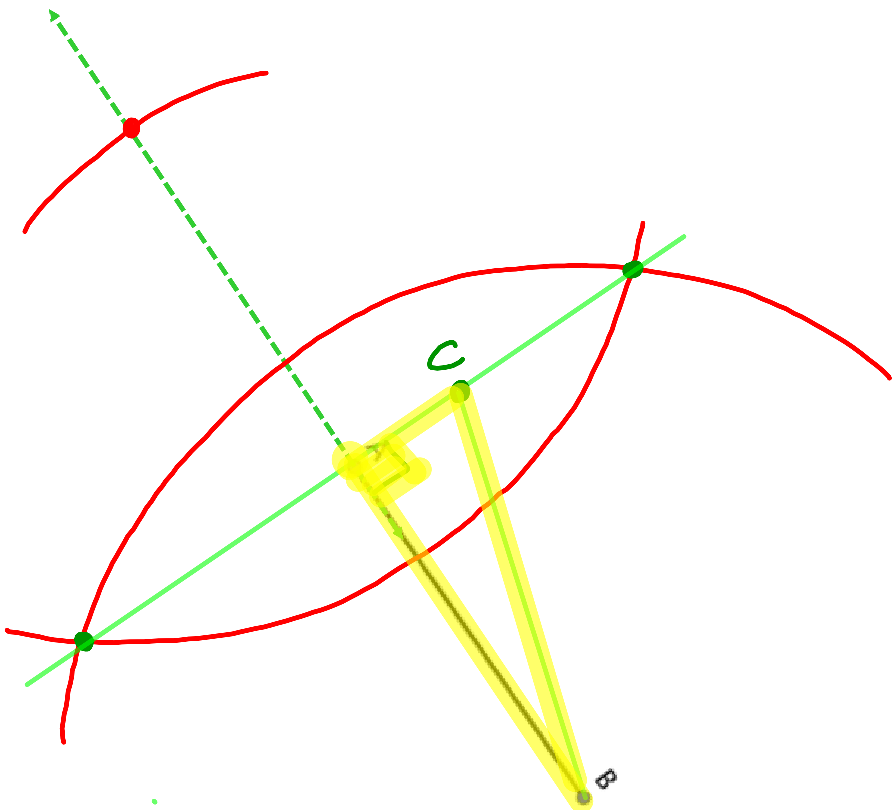
5) Using the given line segment, $\overline{AB}$, construct ΔABC so that $\angle CAB$ is a right angle. Then, use a protractor to measure the angles of your triangle. Verify that $\angle ABC$ and $\angle BCA$ are complementary angles.

A.



$m\angle CAB =$________ $m\angle ABC =$________ $m\angle BCA =$________

B.



$m\angle CAB =$________ $m\angle ABC =$________ $m\angle BCA =$________