Quadratics 4c - Complex Numbers

Standards: N-CN.1, N-CN.2, N-CN.7

Math Practice: Attend to Precision

GLO: #3 Complex Thinker **HW#16**

Learning Targets:
How do you use imaginary numbers to simplify radicals?
How do you add, subtract, & multiply complex numbers?

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Irrational Numbers (Review)

Irrational numbers arise when we take the square root of numbers that are **not** perfect squares .

$\sqrt{13}$

Simplify the following irrational numbers by putting them in simplest radical form. An example is provided.

Example1: Simplify $\sqrt{175}$

Method 1:

$$\sqrt{175} = \sqrt{25 \cdot 7}$$
$$= \sqrt{25} \cdot \sqrt{7}$$
$$= 5\sqrt{7}$$

Find the largest perfect square to take out.

Method 2:

175
5 35
5 7

$$\sqrt{175}$$
$$\sqrt{\underline{5 \cdot 5} \cdot 7}$$
$$\sqrt{5^2 \cdot 7}$$
$$\cancel{\sqrt{5^{2}}} \cdot \sqrt{7} \rightarrow 5\sqrt{7}$$

Factor completely and take out any pairs.

Check your answer with a calculator!

$\sqrt{175}$ ≈ 13.228

$5\sqrt{7}$ ≈ 13.228

Practice Problems – Write in simplest radical form.

1) $\sqrt{18}$ #2

18 → 6, 3; 6 → 3, 2

$\sqrt{3\cdot3\cdot2}$

$\sqrt{3^2\cdot2}$

$\cancel{\sqrt{3^2}}\cdot\sqrt{2}$

$\boxed{3\sqrt{2}}$

2) $3\sqrt{40}$ #1

$3\sqrt{4\cdot10}$

$3\cdot\sqrt{4}\cdot\sqrt{10}$

$3\cdot2\cdot\sqrt{10}$

$6\sqrt{10}$

3) $\sqrt{500}$ #2

500 → 5, 100; 100 → 2, 50; 50 → 2, 25; 25 → 5, 5

$\sqrt{5\cdot5\cdot5\cdot2\cdot2}$

$\sqrt{5^2\cdot5\cdot2^2}$

$\cancel{\sqrt{5^2}}\cdot\sqrt{5}\cdot\cancel{\sqrt{2^2}}$

$5\cdot\sqrt{5}\cdot2$

$\boxed{10\sqrt{5}}$

4) $\sqrt{108}$ #2

108 → 2, 54; 54 → 2, 27; 27 → 9, 3; 9 → 3, 3

$\sqrt{2\cdot2\cdot3\cdot3\cdot3}$

$\sqrt{2^2\cdot3^2\cdot3}$

$\cancel{\sqrt{2^2}}\cdot\cancel{\sqrt{3^2}}\cdot\sqrt{3}$

$2\cdot3\cdot\sqrt{3}$

$6\sqrt{3}$

5) $-2\sqrt{27}$

27 → 3, 9; 9 → 3, 3

$-2\cdot\sqrt{3\cdot3\cdot3}$

$-2\cdot\sqrt{3^2\cdot3}$

$-2\cdot\cancel{\sqrt{3^2}}\cdot\sqrt{3}$

$-2\cdot3\cdot\sqrt{3}$

$-6\sqrt{3}$

6) $\sqrt{720}$

720 → 20, 36; 20 → 4, 5; 36 → 6, 6; 4 → 2, 2; 6 → 2, 3; 6 → 2, 3

$\sqrt{2\cdot2\cdot2\cdot2\cdot3\cdot3\cdot5}$

$\sqrt{2^2\cdot2^2\cdot3^2\cdot5}$

$\cancel{\sqrt{2^2}}\cdot\cancel{\sqrt{2^2}}\cdot\cancel{\sqrt{3^2}}\cdot\sqrt{5}$

$2\cdot2\cdot3\cdot\sqrt{5}$

$\boxed{12\sqrt{5}}$

Complex Numbers

Solve: $x^2 + 1 = 0$

$-1 \qquad -1$

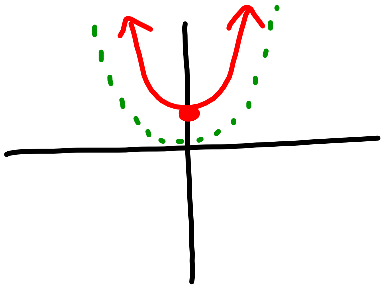
$\sqrt{x^2} = \sqrt{-1}$

"error"

What does the graph look like?

$x^2 + 1$

- concave up
- same width
- vert. shift 1 unit up



What does it show you about the solutions?

No x-intercepts, so no solutions

To solve our problem we will have to use the single complex number:

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$$i = \sqrt{-1}$$

It is often referred to as the **imaginary number** i.

Complex Numbers are written in **standard form** when they are written in the form **a + bi** where "a" and "b" are real numbers and i is the complex square root of -1.

We can write our negative square roots in terms of i using the same algebra of square roots we used before.

Example 2:

$$\sqrt{-4} = \sqrt{(-1)(4)}$$
$$= \sqrt{-1}\sqrt{4}$$
$$= i\sqrt{4}$$
$$= 2i \rightarrow 0+2i$$

Practice Problems - Simplify #1-3.

1) $\sqrt{-9}$

$\sqrt{-1\cdot 9}$

$\sqrt{-1}\cdot\sqrt{9}$

$i\cdot 3$

$3i$

2) $\sqrt{-12}$

$\sqrt{-1\cdot 12}$

$\sqrt{-1}\cdot\sqrt{12}$

$i\cdot 2\sqrt{3}$

$\boxed{2\sqrt{3}\,i}$

3) $2\sqrt{-54}$

$2\sqrt{-1\cdot 54}$

$2\cdot\sqrt{-1}\cdot\sqrt{54}$

$2\cdot i\cdot\sqrt{54}$

$2\cdot i\cdot 3\sqrt{6}$

$\boxed{6\sqrt{6}\,i}$

Solve:

4) $2x^2 + \cancel{45} = -11$

$\quad\;\; \cancel{-45} \quad -45$

$\dfrac{\cancel{2}x^2}{\cancel{2}} = \dfrac{-56}{2}$

$\sqrt{x^2} = \sqrt{-28}$

$\sqrt{-1\cdot 28}$

$\sqrt{-1}\cdot\sqrt{28}$

$i\cdot\cancel{\sqrt{2^2}}\cdot\sqrt{7}$

$\boxed{x = \pm 2\sqrt{7}\,i}$

OR

$\boxed{x = 2\sqrt{7}\,i \;\&\; -2\sqrt{7}\,i}$

5) $3(x+1)^2 = -72$

$\dfrac{\cancel{3}(x+1)^2}{\cancel{3}} = \dfrac{-72}{3}$

$\sqrt{(x+1)^2} = \sqrt{-24}$

$\sqrt{-1}\cdot\sqrt{24}$

$i\cdot\cancel{\sqrt{2^2}}\cdot\sqrt{6}$

$i\cdot 2\cdot\sqrt{6}$

$x + \cancel{1} = \pm 2\sqrt{6}\,i$

$\quad\;\cancel{-1} \quad -1$

$\boxed{x = -1 \pm 2\sqrt{6}\,i}$

OR

$\boxed{x = -1 + 2\sqrt{6}\,i \;\&\; -1 - 2\sqrt{6}\,i}$

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So if $i=\sqrt{-1}$, what is i^2?

$i^2 = \cancel{\sqrt{-1}}^{\cancel{2}}$

$i^2 = -1$

$$i^2 = -1$$

$i^1 = i \Rightarrow \sqrt{-1}$

$i^2 = -1$

What is i^3? $-i$

i^4? 1

$i^7 = -i$

$i^{10} = -1$

Practice Problems – Write the expression as a complex number in standard form.

1) $(-1+2i)+(3+3i)$

$-1+2i+3+3i$

$\boxed{2+5i}$

2) $(2-3i)-(3-7i)$

$2-3i-3+7i$

$\boxed{-1+4i}$

3) $(2+3i)(-6-2i)$

$-12-4i-18i-6i^2$

$-12-22i-6i^2$

$-6\cdot-1$

$-12-22i+6$

$\boxed{-6-22i}$

4) $(3+5i)(3-5i)$

$9-15i+15i-25i^2$

$9-25i^2$

$-25\cdot-1$

$9+25$

34

$34+0i$