

## Rational Functions 3 - Solving Rational Equations

Standards:

A-REI.2

Learning Target:

How do you solve Rational Equations?

**Review:**

Solve:  $x^3 - 25x = 0$

$$x(x^2 - 25) = 0$$

$$x(x+5)(x-5) = 0$$

$$x=0 \quad x+5=0 \quad x-5=0$$

$$\boxed{x=0, 5, -5}$$

There are a few methods you can use to solve rational equations. Always be aware of excluded values of  $x$ .

### Method 1: Cross Multiply

(erase to show)

You can only use this method if each side of the equation has ONE fraction!

#### Example 1 - Solve:

$$a) \frac{3}{x^2 + 4x} \times \frac{1}{x + 4}$$

$$3(x+4) = 1(x^2+4x)$$

$$3x+12 = x^2+4x$$

$$\begin{array}{r} -3x \quad -12 \\ \hline \end{array}$$

$$0 = x^2 + x - 12$$

$$0 = (x-3)(x+4)$$

$$x-3=0 \quad x+4=0$$

$$\boxed{x=3} \quad x=-4$$

$$b) \frac{6}{2x^2 + 2x} \times \frac{x-2}{x+1}$$

$$6(x+1) = (x-2)(2x^2+2x)$$

$$6x+6 = 2x^3+2x^2-4x^2-4x$$

$$\begin{array}{r} -6x \quad -6 \\ \hline \end{array}$$

$$0 = 2x^3 - 2x^2 - 10x - 6$$

$$0 = 2(x^3 - x^2 - 5x - 3)$$

Can not be solved using this method.

Using Method 2

$$\frac{6}{2x^2+2x} = \frac{(x-2) \cdot 2x}{(x+1) \cdot 2x}$$

$$2x(x+1)$$

$$\text{LCD: } 2x(x+1)$$

$$\frac{6 \cancel{2x(x+1)}}{2x(x+1)} = \frac{2x^2 - 4x \cancel{2x(x+1)}}{2x(x+1)}$$

$$6 = 2x^2 - 4x$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$0 = 2x^2 - 4x - 6$$

$$0 = 2(x^2 - 2x - 3)$$

$$0 = 2(x-3)(x+1)$$

$$\cancel{2=0} \quad x-3=0 \quad x+1=0$$

$$\boxed{x=3} \quad x=-1$$

$$\boxed{x=3}$$

**Method 2:** (Will always work)

Rewrite each fraction to have common denominators. Then multiply both sides by the denominators to cancel the denominators. Solve the remaining equation.

**Example 2: Solve**

$$\text{a) } \textcircled{2} \quad \begin{array}{c} 3 \cdot 3 \quad 4 \cdot x \\ \frac{\quad}{3 \cdot 4} + \frac{\quad}{4 \cdot 3} = \frac{5x}{12} \end{array}$$

① LCD: 12

$$\textcircled{3} \quad 12 \left( \frac{9}{12} + \frac{4x}{12} \right) = \left( \frac{5x}{12} \right) 12$$

$$\frac{9 \cdot \cancel{12}}{\cancel{12}} + \frac{4x \cdot \cancel{12}}{\cancel{12}} = \frac{5x \cdot \cancel{12}}{\cancel{12}}$$

$$9 + 4x = 5x$$

$$\quad \quad \quad -4x \quad -4x$$

$$1x = 9$$

$$\boxed{x = 9}$$

$$\text{b) } \textcircled{2} \quad \begin{array}{c} 2 \cdot 3 \quad x \cdot 1 \\ \frac{\quad}{2 \cdot x} - \frac{\quad}{x \cdot 2} = \frac{12 \cdot 2}{x \cdot 2} \end{array}$$

① LCD: 2x

$$\textcircled{3} \quad \frac{6}{2x} - \frac{1x}{2x} = \frac{24}{2x}$$

$$6 - 1x = 24$$

$$\quad \quad \quad -6 \quad \quad -6$$

$$-x = 18$$

$$\quad \quad \quad +1 \quad \quad +1$$

$$\boxed{x = -18}$$

**Example 3: Solve**

Always check your answers!

a) ②

$$\frac{5x}{(x+1)} = \frac{4}{\cancel{(x+1)}} - \frac{5}{(x+1)}$$

① LCD: (x+1)

$$\textcircled{3} \quad \frac{5x}{\cancel{(x+1)}} = \frac{4x+4}{\cancel{(x+1)}} - \frac{5}{\cancel{(x+1)}}$$

$$5x = 4x + 4 - 5$$

~~-4x~~   ~~-4x~~

$$x = -1$$

No Solution

**Example 3: Solve**

$$\text{b) } \frac{(x+2) \cdot (3x-2)}{(x+2) \cdot \underline{(x-2)}} = \frac{6}{\underline{x^2-4}} + \frac{1}{1} \cdot \frac{(x-2)(x+2)}{(x-2)(x+2)}$$

$$\text{① LCD: } \underline{(x-2)} \underline{(x+2)}$$

$$\text{③ } \frac{3x^2 - 2x + 6x - 4}{\cancel{(x-2)(x+2)}} = \frac{6}{(x+2)(x-2)} + \frac{x^2 - 2x + 2x - 4}{\cancel{(x+2)(x-2)}}$$

$$3x^2 + 4x - 4 = 6 + x^2 - 4$$

$\begin{array}{ccc} -x^2 & & -x^2 \\ -2 & & -2 \end{array}$

$$2x^2 + 4x - 6 = 0$$

$$2(x^2 + 2x - 3) = 0$$

$$2(x+3)(x-1) = 0$$

$$\underline{x = -3} \quad \underline{x = 1}$$

$$\boxed{x = -3, 1}$$

Extra Practice - Solve

a)

$$\frac{16}{\cancel{x^2 - 16}} + \frac{3}{1} = \frac{(2x+1)(x+4)}{(x-4)(x+4)}$$

(+4)(-4)  
(+4)(-4)

$$\text{LCD: } (x+4)(x-4)$$

$$\frac{16}{(x+4)(x-4)} + \frac{3x^2-48}{(x+4)(x-4)} = \frac{2x^2+9x+4}{(x+4)(x-4)}$$

$$16 + 3x^2 - 48 = 2x^2 + 9x + 4$$

$$3x^2 - 32 = 2x^2 + 9x + 4$$

-2x^2   -4   -2x^2   -4  
-9x

$$x^2 - 9x - 36 = 0$$

$$(x-12)(x+3) = 0$$

$$x-12=0 \quad x+3=0$$

$$\boxed{x=12} \quad \boxed{x=-3}$$

$$\boxed{x=12, -3}$$

b)

$$(x+2) \cdot \frac{2(x-2)}{x^2-10x+16} = \frac{2}{(x+2)} \cdot (x-8)(x-2)$$

$$(x+2) \cdot (x-8)(x-2)$$

LCD:  $(x-8)(x-2)(x+2)$

$$(x+2)(2x-4)$$

$$2(x^2-10x+16)$$

	x	+2
2x	2x <sup>2</sup>	+4x
-4	-4x	-8

$$\frac{2x^2-8}{(x-8)(x-2)(x+2)} = \frac{2x^2-20x+32}{(x-8)(x-2)(x+2)}$$

$$2x^2-8 = 2x^2-20x+32$$

$$0 = -20x + 40$$

$$0 = -20(x-2)$$

$$-20 = 0 \quad x-2 = 0$$

$$x = 2$$

No Solution



$$c) \quad \frac{(x+6) \cdot 2}{(x+6) \cdot 3x} + \frac{2}{3} = \frac{8}{(x+6) \cdot 3x}$$

$\begin{matrix} x(x+6) \\ x(x+6) \end{matrix}$

LCD:  $3x(x+6)$

$$\frac{2x+12}{3x(x+6)} + \frac{2x^2+12x}{3x(x+6)} = \frac{24x}{3x(x+6)}$$

$$2x+12 + 2x^2+12x = 24x$$

$$2x^2 + 14x + 12 = 24x$$

$\begin{matrix} -24x & -24x \end{matrix}$

$$2x^2 - 10x + 12 = 0$$

$$2(x^2 - 5x + 6) = 0$$

$$2(x-2)(x-3) = 0$$

~~$2=0$~~   $x-2=0$   $x-3=0$

$$\underline{x=2} \quad \underline{x=3}$$

$$\boxed{x=2, 3}$$

$$d) (x-4) \cdot \frac{x}{2x-6} = \frac{2}{(x-4)} \cdot 2(x-3)$$

$$(x-4) \cdot 2(x-3)$$

$$LCD = 2(x-3)(x-4)$$

$$\frac{x^2-4x}{2(x-3)(x-4)} = \frac{4x-12}{2(x-3)(x-4)}$$

$$x^2-4x = 4x-12$$

$$\begin{array}{r} -4x \quad -4x \\ +12 \quad +12 \end{array}$$

$$x^2-8x+12=0$$

$$(x-6)(x-2)=0$$

$$x-6=0 \quad x-2=0$$

$$\underline{x=6} \quad \underline{x=2}$$

$$\boxed{x=2,6}$$

$$e) \quad \frac{2 \cdot 2(x+7)}{2 \cdot (x+4)} - \frac{2}{1} = \frac{2x+20}{2(x+4)}$$

*(Red annotations: 2(x+4) above the first fraction, 2(x+4) below the denominator of the second fraction, and 2x+8 crossed out in the numerator of the right side.)*

① LCD:  $2(x+4)$

$$\textcircled{3} \quad \frac{4x+28}{2(x+4)} - \frac{4x+16}{2(x+4)} = \frac{2x+20}{2(x+4)}$$

*(Red annotations: Red arrows pointing from the 28 in the first numerator to the 16 in the second numerator, and the entire equation is crossed out with a red line.)*

$$4x+28-4x-16=2x+20$$

$$12=2x+20$$

*(Red annotations: -20 below 12, -20 below 20, and the entire equation is crossed out with a red line.)*

$$\frac{-8}{2} = \frac{2x}{2}$$

*(Red annotations: -8 below 12, 2 below 2, and the entire equation is crossed out with a red line.)*

$$x = -4$$

*(Red annotations: The entire equation is crossed out with a red line.)*

No Solution