Rational Functions 2b: Add & Subtract with Unlike Denominators

Standards:A-APR.7, A-SSE.1a

GLO: #3 Complex Thinker

Math Practice: #7 - Look for & make use of Structure

Learning Targets

- How do you find the Least Common Denominator?
- How do you change fractions to have the same denominator?

Review:

Simplify: $\frac{2}{3} - \frac{1}{2}$

$\frac{2\cdot 2}{2\cdot 3} - \frac{1\cdot 3}{2\cdot 3}$

$\frac{4}{6} - \frac{3}{6}$

$\frac{4-3}{6}$

$\frac{1}{6}$

$\frac{1}{\cancel{18}} - \frac{1}{\cancel{30}}$

$18 = 2\cdot 9 = 2\cdot 3\cdot 3$

$30 = 10\cdot 3 = 5\cdot 2\cdot 3$

$\frac{5\cdot 1}{5\cdot 2\cdot 3\cdot 3} - \frac{1\cdot 3}{2\cdot 3\cdot 5\cdot 3}$

$\frac{5}{2\cdot 3\cdot 3\cdot 5} - \frac{3}{2\cdot 3\cdot 3\cdot 5}$

$\frac{5-3}{2\cdot 3\cdot 3\cdot 5}$

$\frac{\cancel{2}}{\cancel{2}\cdot 3\cdot 3\cdot 5} \rightarrow \frac{1}{45}$

When adding or subtracting rational numbers such as 1/18 and 1/30 , you must first find the **least common multiple (LCM)** of the denominator, because rational numbers cannot be added/subtracted if they do not have the same denominator. The same is true for rational expressions.

Finding the Least Common Mulitple (LCM)

1. Completely factor each monomial or polynomial. ~~If a factor is repeated, use an exponent to indicate this~~.
2. To determine the LCM, bring out what the denominators have in common, then bring out anything leftover.

Example 1: Find the least common multiple

$k^3 - k^2 - 12k$ $\qquad$ $k^4 - 8k^3 + 16k^2$

$k(k^2-k-12)$ $\qquad$ $k^2(k^2-8k+16)$

Factor: $k(k+3)(k-4)$ $\qquad$ $k^2(k-4)(k-4)$

$k \cdot k(k-4)(k-4)(k+3)$

For each polynomial, list out the factors in a row. Keep the factors aligned in columns, with a different column for each factor (whether it is a monomial or a binomial).

Factors of $k^3 - k^2 - 12k$: $\quad k \quad k+3 \quad k-4$

Factors of $k^4 - 8k^3 + 16k^2$: $\quad k \cdot k \quad (k-4)(k-4)$

To determine the LCM, look in each column and select the term they have in common, add in the leftover terms, then multiply the terms together.

(erase to show)

LCM: $k\,k(k-4)(k+3)(k-4)$

$k^2(k-4)^2(k+3)$

Practice: Find the least common multiple of

$(q-2)(q-2)$ and $q(q-1)(q-2)$

LCM: $(q-2)\ (q-2)\ q\ (q-1)$

or

$\boxed{q(q-2)(q-2)(q-1)}$

$\boxed{q(q-2)^2(q-1)}$

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<u>Adding and Subtracting Rational Expressions:</u>

1. Find the least common multiple (LCM) of the denominators. (Also known as the **least common denominator - LCD**)

2. For each rational expression, find an equivalent expression that has the LCM as its denominator. (Multiply the denominator by its "missing" factors. Multiply the numerator by the same factors.)

3. If necessary, simplify each numerator by multiplying terms together.

4. Add or subtract across the numerator only. The LCM will remain as the denominator.
   If subtracting, be sure to distribute the negative sign to each term in the second expression's numerator.

5. If possible, factor the numerator completely. If any factors cancel from the numerator and the denominator, simplify.

6. Find the domain restriction (excluded values): Set each factor from the **LCM ≠ 0** and solve.

Example 2: Simplify

② $\frac{(x-2)\cdot 3}{(x-2)\cdot(x+1)} + \frac{x\cdot(x+1)}{(x-2)\cdot(x+1)}$

① LCD: $(x+1)\ (x-2)$

③ $\frac{3x-6}{(x+1)(x-2)} + \frac{x^2+x}{(x+1)(x-2)}$

④ $\frac{x^2+4x-6}{(x+1)(x-2)}$ ← cannot be factored

$a=1 \quad b=4 \quad c=-6$

$a\cdot c=-6 \quad b=4$

$-1 \quad 6$

$-2 \quad 3$

$$\boxed{\frac{x^2+4x-6}{(x+1)(x-2)}}$$

Domain Restriction:
(Excluded Value)

$(x+1)(x-2)\neq 0$

$x+1\neq 0 \quad x-2\neq 0$

$x\neq -1 \quad x\neq 2$

$\boxed{x\neq -1, 2}$

Answer

Example 3: Simplify

Domain Restriction: (Excluded Value)

2
$$\frac{x-1}{\cancel{x^2+4x+3}} - \frac{1}{(x+1)} \cdot (x+3)$$

$\cdot (x+3)$

$(x+3)(x+1)$

$(x+1)(x+3) \neq 0$

$x+1 \neq 0 \quad x+3 \neq 0$

$\boxed{x \neq -1, -3}$

1 LCD: $(x+1)(x+3)$

3 $$\frac{x-1}{(x+3)(x+1)} - \frac{x+3}{(x+3)(x+1)}$$

4 $$\frac{x-1-x-3}{(x+3)(x+1)}$$

$$\boxed{\frac{-4}{(x+3)(x+1)}}$$

Answer

Practice: Simplify and state the domain restriction

1. ② $\frac{(x+2)\cdot 1}{(x+2)\cdot x} + \frac{1 \cdot x}{(x+2) \cdot x}$

DR/EV:

$x(x+2) \neq 0$

$x \neq 0 \quad x+2 \neq 0$

$\boxed{x \neq -2, 0}$

① LCD: $x(x+2)$

③ $\frac{x+2}{x(x+2)} + \frac{x}{x(x+2)}$

④ $\frac{x+2+x}{x(x+2)}$

$\frac{2x+2}{x(x+2)}$

⑤ $\boxed{\frac{2(x+1)}{x(x+2)}}$

Answer

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Practice: Simplify and state the domain restriction

2. ② $\frac{(4x+5)\cdot(x+6)}{(4x+5)\cdot(x+1)}+\frac{x\cdot(x+1)}{(4x+5)\cdot(x+1)}$

DR/EV:

$(x+1)(4x+5)\neq 0$

$x+1\neq 0 \quad 4x+5\neq 0$

$x\neq -1 \quad 4x\neq -5$

$x\neq -\frac{5}{4}$

$\boxed{x\neq -1, -\frac{5}{4}}$

① LCD: $(x+1)(4x+5)$

③ $\frac{4x^2+29x+30}{(x+1)(4x+5)}+\frac{x^2+x}{(x+1)(4x+5)}$

④ $\frac{4x^2+29x+30+x^2+x}{(x+1)(4x+5)}$

$\frac{5x^2+30x+30}{(x+1)(4x+5)} \leftarrow 5(x^2+6x+6)$

$a=1 \quad b=6 \quad c=6$

$a\cdot c=6 \quad b=6$

1 6

2 3

⑤ $\boxed{\frac{5(x^2+6x+6)}{(x+1)(4x+5)}}$

Answer

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Practice: Simplify and state the domain restriction

②

3. $\frac{2x}{2x} \cdot \frac{(9x+8)}{(x-7)} - \frac{(3x-6)}{2x} \cdot \frac{(x-7)}{(x-7)}$

DR/EV:

$2x(x-7) \neq 0$

$\frac{2x}{2} \neq \frac{0}{2}$ $\quad x-7 \neq 0$, $+7 \quad +7$

$\boxed{x \neq 0, 7}$

① LCD: $2x(x-7)$

③ $\frac{18x^2+16x}{2x(x-7)} - \frac{3x^2-27x+42}{2x(x-7)}$

④ $\frac{18x^2+16x-3x^2+27x-42}{2x(x-7)}$

$\frac{15x^2+43x-42}{2x(x-7)}$ ← not factorable

$a=15 \quad b=43 \quad c=-42$

$a \cdot c = -630$

~~-1 630~~
~~-2 315~~
~~-3 210~~
~~-5 126~~
~~-6 105~~
~~-7 90~~
~~-9 70~~
~~-10 63~~
~~-14 45~~
~~-15 42~~
~~-18 35~~
~~-21 30~~

⑤ $\boxed{\frac{15x^2+43x-42}{2x(x-7)}}$

Answer

Practice: Simplify and state the domain restriction

② $(x-5)\cdot$

4. $$\frac{x}{\cancel{2x+2}} - \frac{3}{\cancel{x^2-4x-5}} \cdot 2$$

$(x-5)\cdot \quad 2(x+1) \qquad (x+1)(x-5) \cdot 2$

① LCD: $2\,(x+1)\,(x-5)$

③ $$\frac{x^2-5x}{2(x+1)(x-5)} - \frac{6}{2(x+1)(x-5)}$$

④ $$\frac{x^2-5x-6}{2(x+1)(x-5)}$$

⑤ $$\frac{(x-6)\cancel{(x+1)}}{2\cancel{(x+1)}(x-5)}$$

$$\boxed{\frac{(x-6)}{2(x-5)}}$$

DR/EV:

$2(x+1)(x-5) \neq 0$

$\cancel{2 \neq 0} \quad x+1 \neq 0 \quad x-5 \neq 0$

$\boxed{x \neq -1, 5}$

Answer

Practice: Simplify and state the domain restriction

5. ② $\dfrac{(x-3)\cdot(x+1)}{\cancel{x^2+6x+9}} - \dfrac{1}{\cancel{x^2-9}} \cdot (x+3)$

$(x-3)\cdot \quad (x+3)(x+3) \qquad (x+3)(x-3) \cdot (x+3)$

① LCD: $(x+3)\ (x+3)\ (x-3)$

③ $\dfrac{x^2-2x-3}{(x+3)(x+3)(x-3)} - \dfrac{x+3}{(x+3)(x+3)(x-3)}$

④ $\dfrac{x^2-2x-3-x-3}{(x+3)(x+3)(x-3)}$

$\boxed{\dfrac{x^2-3x-6}{(x+3)(x+3)(x-3)}}$

DR/EV:

$(x+3)(x+3)(x-3) \neq 0$

$x+3 \neq 0 \quad x+3 \neq 0 \quad x-3 \neq 0$

$\boxed{x \neq -3, 3}$

Answer

(last slide)

Practice: Simplify and state the domain restriction

6. ② $4 \cdot \dfrac{(x-1)}{\cancel{3x^2+8x+5}} - \dfrac{(x-1)}{\cancel{12x+20}} \cdot (x+1)$

$4 \cdot (3x+5)(x+1) \quad 4(3x+5) \cdot (x+1)$

① LCD: $4(3x+5)(x+1)$

DR/EV:

$4(3x+5)(x+1) \neq 0$

$\cancel{4 \neq 0} \quad 3x+5 \neq 0 \quad x+1 \neq 0$

$3x \neq -5 \quad x \neq -1$

$x \neq -\frac{5}{3}$

③ $\dfrac{4x-4}{4(3x+5)(x+1)} - \dfrac{x^2-1}{4(3x+5)(x+1)}$

④ $\dfrac{-x^2+4x-3}{4(3x+5)(x+1)} \rightarrow -1(x^2-4x+3)$

$\boxed{x \neq -1, -\frac{5}{3}}$

⑤ $\dfrac{-(x-3)(x-1)}{4(3x+5)(x+1)}$

Answer