Quadratics 4b - Vertex Form

Standard: F-IF.7

Math Practice: Look for and make use of structure

GLOs: #3-Complex Thinker

HW: WS#15

Learning Target:

- How does Vertex Form help you determine the transformations of a quadratic function?

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(erase to show)

In previous lessons we learned how to graph simple translations of the parent quadratic function defined by $f(x) = x^2$. $ax^2 + c$

We began with functions of the form **$g(x) = x^2 + k$**. If the value of k is positive the parent functions simply shifts up - **k** units and if the value of k is **negative** the parent function shifts down k units.

Now let us consider quadratic functions of the form **$g(x) = (x - h)^2$**. If the value of h is **positive** then the parent function f moves to the right h units. If the value of h is negative, f moves to the **left** h units.

We now combine the two types of functions { $f(x) = x^2 + k$ and $f(x) = (x-h)^2$ } and consider quadratic functions written in the form

$$g(x) = a(x-h)^2 + k$$

In this case the value of h reveals how much the parent function f moves in the horizontal direction and the value of k reveals how far f moves in the vertical direction. **In particular, the vertex of *f* moves to the point (h, k).** Consequently, we refer to this form of a quadratic function as **Vertex Form**.

Note: the role that a plays is the same as before.

The quadratic function is said to be in **vertex form** if it is written as

$$f(x) = a(x-h)^2 + k$$

The vertex of the graph of the quadratic function is at the point **(h, k)**.

We have three distinct forms commonly used to describe quadratic functions:

(write in)

i. Vertex Form $f(x)=a(x-h)^2+k$

ii. Standard Form $f(x)=ax^2+bx+c$

iii. Factored Form $f(x)=a(x-s)(x-t)$

Reflections: Which form of quadratic functions is easier to use when you are trying to locate:

a. The x-intercepts? Factored Form
$(s,0)$ & $(t,0)$

b. The y – intercept? Standard Form
$(0,c)$

c. The vertex? Vertex Form
(h,k)

The purpose of this lesson is to investigate the vertex form of a quadratic function, $g(x) = a(x - h)^2 + k$

The vertex form is very informative, in that it can be used to:

(a) Describe the transformations of the parent function $f(x) = x^2$

(b) Identify the vertex of a parabola (*h*, *k*);

(c) Determine the maximum or minimum value of a quadratic function;

(d) Determine the domain and range of a quadratic function; and

(e) Sketch the graph of a parabola (find the x-intercepts, if they exisit).

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Example 1: Given $g(x)=2(x-1)^2+3$

a) Describe how the parent function $f(x)=x^2$ is transformed.

b) Identify the vertex of this parabola.

c) Determine if the function has a maximum or minimum, and its value.

d) State the domain and range of the function

e) Sketch the graph. Find the x-intercepts of the parabola, if there are any.

Example 1: Given $g(x)=2(x-1)^2+3$

Solution:

a) Describe how the parent function f(x)=x2 is transformed. a = 2, h = 1, k = 3

- No reflection across the x-axis
- Narrower
- Horizontal shift 1 unit right
- Vertical shift 3 units up

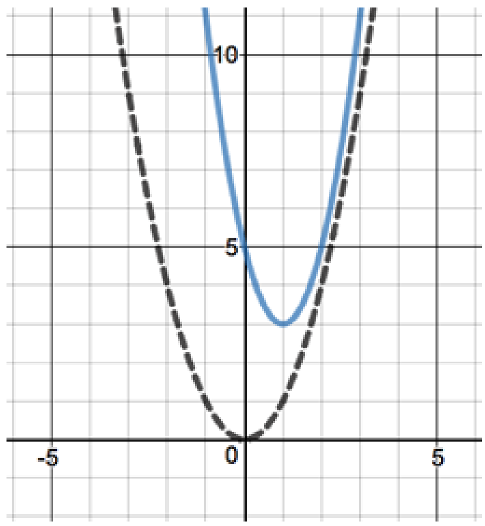
b) Vertex (1, 3)

c) Minimum value 3

d) Domain: All Real Numbers

Range: $g(x)\geq 3$

e) sketch: **(no x-intercepts)**



Example 2: Given $g(x) = -\frac{1}{2}(x+4)^2 + 1$

a) Describe how the parent function $f(x)=x^2$ is transformed.

b) Identify the vertex of this parabola.

c) Determine if the function has a maximum or minimum, and its value.

d) State the domain and range of the function

e) Sketch the graph. Find the x-intercepts of the parabola, if there are any.

Solution:

$a = -\frac{1}{2}$, $h = -4$, $k = 1$

reflected over x-axis (concave down)

vertical shift 1 unit up

horizontal shift 4 units down

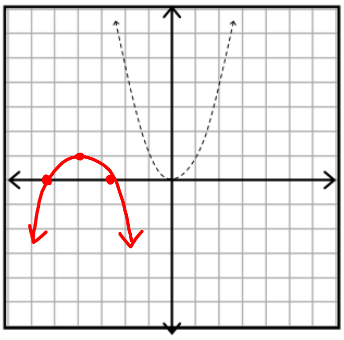
wider

Vertex: (-4, 1)

max value 1

Domain: all real #s

Range: $g(x) \le 1$



x-int: (set y=0 & solve)

$0 = -\frac{1}{2}(x+4)^2 + \cancel{1}$ ① isolate $(x+4)^2$

$-1 \quad \cancel{-1}$

$-2(-1) = \left(-\cancel{\tfrac{1}{2}}(x+4)^2\right) \cdot -\cancel{2}$

$\sqrt{2} = \sqrt{(x+4)^{\cancel{2}}}$ ② square root both sides

$\pm 1.414 = x+4$

$x+\cancel{4} = 1.414$ $\quad$ $x+\cancel{4} = -1.414$

$\cancel{-4} \quad -4$ $\quad$ $\cancel{-4} \quad -4$

$x = -2.586$ $\quad$ $x = -5.414$

$\boxed{(-2.6, 0) \quad (-5.4, 0)}$

Practice: For each function below:

a. Describe how the parent function f(x) = x^2 is transformed.
b. Identify the vertex of this parabola.
c. Determine if the function has a maximum or minimum, and its value.
d. State the domain and range of the function.
e. Sketch the graph. Find the x-intercepts of the parabola if there are any.

1) $g(x) = -x^2 + 6$ $\rightarrow g(x) = -1(x-0)^2 + 6$

Solution:

$a =$ -1, $h =$ 0, $k =$ 6

reflected over x-axis (concave down)

same width

no horizontal shift

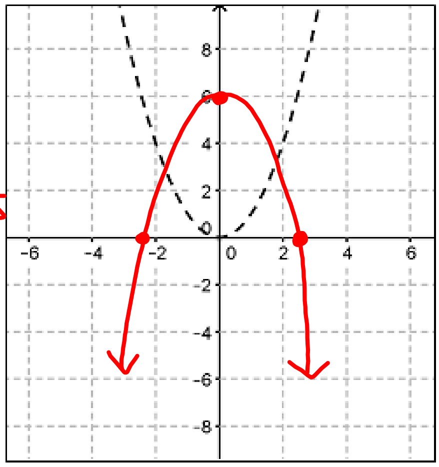
vertical shift 6 units up

Vertex: (0, 6)

max value 6

Domain: all real numbers

Range: $g(x) \le 6$



x-int: (set y=0 & solve)

$0 = -x^2 + 6$
$-6 \quad\quad -6$

$\frac{-6}{-1} = \frac{-x^2}{-1}$

$\sqrt{6} = \sqrt{x^2}$

$x = \pm 2.449$

$\boxed{(2.4, 0)\ (-2.4, 0)}$

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2) $g(x) = 1(x - 5)^2 - 8 \Rightarrow g(x) = 1(x-(-5))^2 + -8$

Solution:

$a =$ 1, $h =$ 5, $k =$ −8

no reflection over x-axis (concave up)

same width

horizontal shift 5 units right

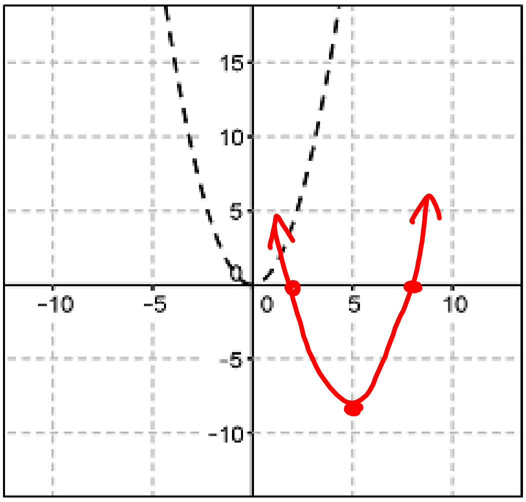
vertical shift 8 units down

Vertex: (5, −8)

min value −8

Domain: all real numbers

Range: $g(x) \geq -8$



x-int: (set y=0 & solve)

$0 = (x-5)^2 - \cancel{8}$

$+8 \qquad \cancel{+8}$

$\sqrt{8} = \sqrt{(x-5)^{\cancel{2}}}$

$x - 5 = \pm 2.828$

$x - \cancel{5} = 2.828 \qquad x - \cancel{5} = -2.828$

$+\cancel{5} \quad +5 \qquad +\cancel{5} \quad +5$

$x = 7.828 \qquad x = 2.172$

$\boxed{(7.8, 0) \qquad (2.2, 0)}$

3) $g(x) = 2(x+8)^2 - 2 \rightarrow g(x) = 2(x-(-8))^2 + -2$

Solution:

$a =$ 2, $h =$ -8, $k =$ -2

no reflection over x-axis (concave up)

narrower

horizontal shift 8 units left

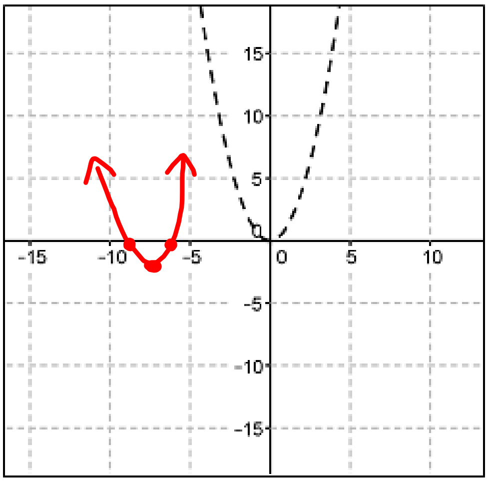
vertical shift 2 units down

Vertex: (-8,-2)

min value -2

Domain: all real numbers

Range: $g(x) \geq -2$



x-int: (set y=0 & solve)

$0 = 2(x+8)^2 - \cancel{2}$

$+2 \qquad +\cancel{2}$

$\frac{2}{2} = \frac{\cancel{2}(x+8)^2}{\cancel{2}}$

$\sqrt{1} = \sqrt{(x+8)^2}$

$\pm 1 = x+8$

$x + \cancel{8} = 1 \qquad x + \cancel{8} = -1$

$-\cancel{8} \quad -8 \qquad -\cancel{8} \quad -8$

$x = -7 \qquad x = -9$

$\boxed{(-7,0) \text{ \& } (-9,0)}$

4) $g(x)=\frac{1}{2}(x+5)^2-16 \rightarrow g(x)=\frac{1}{2}(x-(-5))^2+(-16)$

Solution:

$a=$ $\frac{1}{2}$, $h=$ -5, $k=$ -16

no reflection over x-axis (concave up)

wider

horizontal shift 5 units left

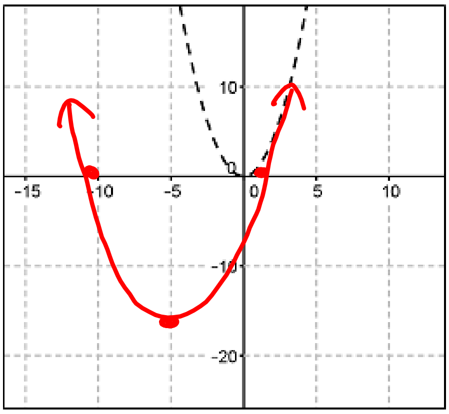
vertical shift 16 units down

Vertex: $(-5,-16)$

min value -16

Domain: all real numbers

Range: $g(x)\geq -16$



x-int: (set y=0 & solve)

$0=\frac{1}{2}(x+5)^2-\cancel{16}$

$+16 \quad +\cancel{16}$

$2\cdot(16)=\left(\frac{1}{\cancel{2}}(x+5)^2\right)\cdot\cancel{2}$

$\sqrt{32}=\sqrt{(x+5)^{\cancel{2}}}$

$x+5=\pm 5.657$

$x+\cancel{5}=5.657$ $\qquad$ $x+\cancel{5}=-5.657$

$-\cancel{5} \quad -5$ $\qquad$ $-\cancel{5} \quad -5$

$x=0.657$ $\qquad$ $x=-10.657$

$\boxed{(0.7,0) \ \& \ (-10.7,0)}$

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