

Module 16c: Volumes of Pyramids & Cones

Math Practice(s):

- Model with mathematics.
- Look for & make use of structure.

Learning Target(s):

- Apply the formulas for volume to solve problems in real-world context.

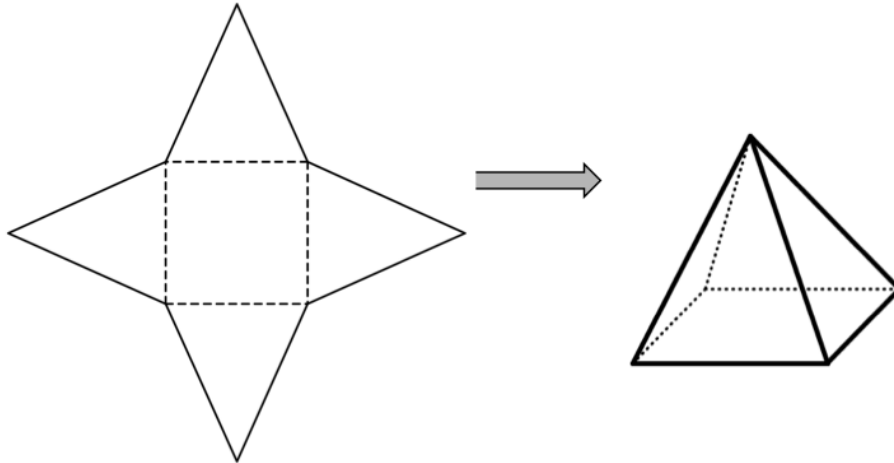
Homework:

HW#10: 16c #1-5

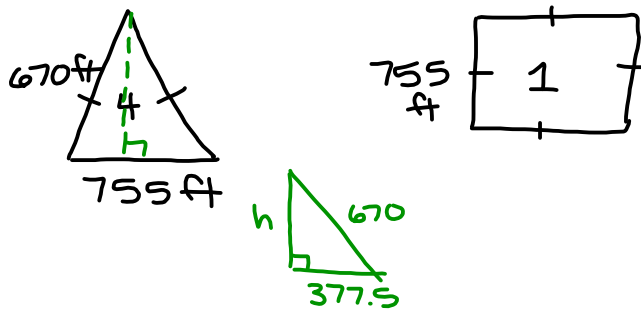
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Warm-up:

- If we cut out the figure on the left (a two-dimensional object, comprised of a square with an isosceles triangle on each side), then fold the triangles upward along the dashed lines, you would end up with the figure on the right: a pyramid with a square base (known as a square pyramid).



The Great Pyramid of Giza in Egypt, is a pyramid with a square base where each side of the base has a length of 755 ft., and the two legs of each triangle have a length of 670 ft. Determine the surface area of the Great Pyramid of Giza.



$$377.5^2 + h^2 = 670^2$$

⋮

$$h = \sqrt{306,393.75}$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(755)(\sqrt{306,393.75})$$

$$= 208,956.9916$$

$$A = 755(755)$$

 × 4

$$835,827.9664 + 570,025$$

$$1,405,852.966$$

$$\boxed{1,405,853 \text{ ft}^2}$$

(erase to show)

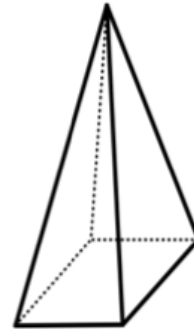
Pyramids (#VOC)

A 3-dimensional figure with a polygon base and
△ faces that meet at the apex.

Volume of a Pyramid (#THM)

$$V = \frac{1}{3}Bh$$

B = area of the base



Example 1: Find the volume of a square pyramid with the given dimensions. Express your answer in exact form and as a decimal rounded to the nearest hundredths place.

a) base length of 3 in. and a pyramid height of 7 in.

$$V = \frac{1}{3}(9)(7)$$

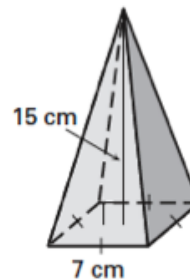
$$V = \frac{1}{3}(63)$$

$$V = 21 \text{ in}^3$$

$$B = 3 \cdot 3$$

$$= 9$$

b)



$$B = 7^2$$

$$= 49$$

$$V = \frac{1}{3}(49)(15)$$

$$= \frac{1}{3}(735)$$

$$V = 245 \text{ cm}^3$$

(erase to show)

Cones (#VOC)

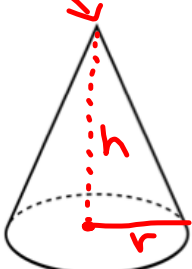
A 3-dimensional figure with a circular base and a single vertex.

Volume of a Cone (#THM)

$$V = \frac{1}{3} Bh$$

$$B = \pi r^2$$

B = area of the base



Example 2: Find the volume of the cone with the given dimensions. Express your answer in exact form and as a decimal rounded to the nearest hundredths place.

a) radius of 4 cm and height of 6 cm

$$V = \frac{1}{3} Bh$$

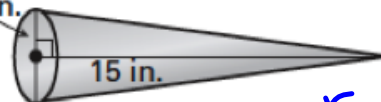
$$B = \pi(4)^2 = 16\pi$$

$$= \frac{1}{3}(16\pi)(6)$$

$$V = 32\pi \text{ cm}^3$$

$$V \approx 100.53 \text{ cm}^3$$

b) 4 in.



$$V = \frac{1}{3}(4\pi)(15)$$

$$r = 2 \text{ in}$$

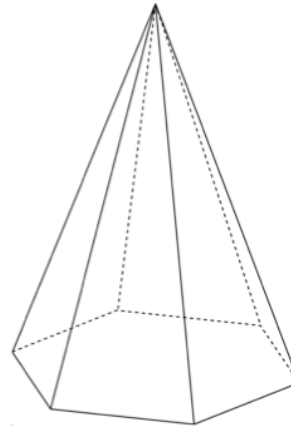
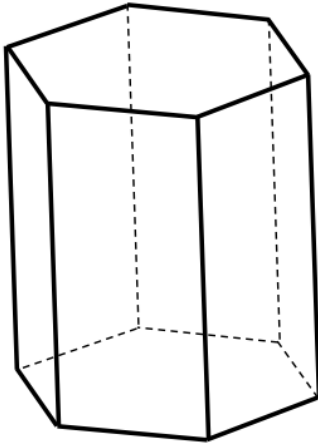
$$B = \pi(2)^2 = 4\pi$$

$$V = 20\pi \text{ in}^3$$

$$V \approx 62.83 \text{ in}^3$$

Example 3: A hexagonal prism and a hexagonal pyramid are shown below.

- The base of both figures are congruent hexagons with an area of 75 square feet.
- The height of both figures is the same length.



- A. If the height of both figures is 12 feet, determine the volume of both figures.

$$V_{\text{prism}} = Bh$$

$$= 75(12)$$

$$V = 900 \text{ ft}^3$$

$$V_{\text{pyramid}} = \frac{1}{3}(900)$$

$$V = 300 \text{ ft}^3$$

- B. Determine the height of the hexagonal prism if its volume is 225 cubic feet. Set-up and solve an equation to show how you determined your answer.

$$V = Bh$$

$$225 = \frac{75}{75}h$$

$$h = 3 \text{ ft}$$

- C. Determine the height of the hexagonal pyramid if its volume is 250 cubic feet. Set-up and solve an equation to show how you determined your answer.

$$250 = \frac{1}{3}(75)h$$

$$250 = \frac{25}{25}h$$

$$h = 10 \text{ ft}$$

Practice:

Grandma's birthday is in a few weeks and we would like to buy her a new fish tank for her pet fish, Ramsey and Karyl. Ramsey and Karyl like a lot of water to swim in, and therefore Grandma needs a sufficiently large fish tank. There are two tanks available, and we want to purchase the one that holds the most water. The first tank is a cylinder with diameter 12 cm and height 30 cm. The second tank is a cone with base diameter 18 cm and height 35 cm.

1. Which tank should we purchase?

Cylinder

$$V = Bh \quad B = \pi(6)^2$$

$$= (36\pi)(30) = 36\pi$$

$$= 1080\pi \text{ cm}^3$$

Cone

$$V = \frac{1}{3}Bh \quad B = \pi(9)^2$$

$$= \frac{1}{3}(81\pi)(35) = 81\pi$$

$$= 945\pi \text{ cm}^3$$

We should purchase the cylindrical tank.

2. How much more water does the larger tank hold than the smaller tank?

$$1080\pi - 945\pi$$

$$135\pi$$

The larger tank holds about 424 cm^3 more water.

3. Suppose the diameter of each tank decreases by 2 cm. Which tank would then hold the most water?

Cylinder

$$d = 10 \text{ cm}, r = 5 \text{ cm}$$

$$h = 30 \text{ cm}$$

$$B = \pi(5)^2$$

$$V = (25\pi)(30) = 25\pi$$

$$= 750\pi \text{ cm}^3$$

$$\approx 2356.2 \text{ cm}^3$$

Cone

$$d = 16 \text{ cm}, r = 8 \text{ cm}$$

$$h = 35 \text{ cm}$$

$$B = \pi(8)^2$$

$$V = \frac{1}{3}(64\pi)(35) = 64\pi$$

$$= \frac{2240}{3}\pi \text{ cm}^3$$

$$\approx 2345.7 \text{ cm}^3$$

The cylindrical tank would still hold more water.