Module 15e: Area of a Sector of a Circle

Math Practice(s):

-Construct viable arguments & critique the reasoning of others.

-Look for & make use of structure.

Learning Target(s):

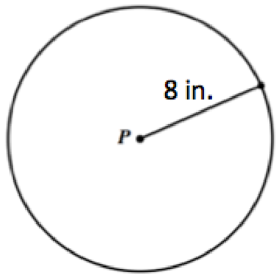
- Understand & apply the proportional relationship between the area of a sector of a circle & the central angle of the sector.

Homework:

HW#7: 15e #1-6

Nov 20-12:32 PM

Warm-up



1. Circle P has a radius of 8 inches. Determine its area. Express your answer in exact form and as a decimal rounded to the nearest hundredths place.

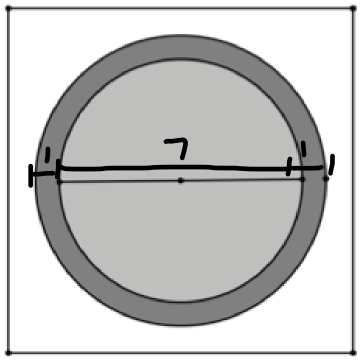
$A=\pi r^2 = \pi(8)^2 = \boxed{64\pi \text{ in}^2 \approx 201.06 \text{ in}^2}$

2. A pizza restaurant is offering two specials: a circular pizza with a 14-inch diameter and a square pizza with a side measuring 14 inches. Which pizza covers a larger area? How many more square inches does the larger pizza cover? Express your answer as a decimal rounded to the nearest hundredths place.

$\square = 14^2 = 196 \text{ in}^2$ $\quad \bigcirc = 7^2 \cdot \pi = 153.94 \text{ in}^2$ $\quad 196 - 153.94 = 42.06$

The square pizza is larger by 42.06 in².

3. The official wrestling mat being used for a high school tournament has the following dimensions:



- The inner circle has a diameter of 7 meters. This is the main area of the wrestling mat.
- Around the inner circle is a circular ring that maintains a 1-meter distance from the inner circle all the way around. This circular ring is referred to as the "red zone" of the mat.
- The rings are centered on a square mat that measures 12 meters on each side. The area of the mat around the red zone is referred to as the "protection zone".

A. Determine the area of the circular ring referred to as the "red zone". Express your answer in exact form and as a decimal rounded to the nearest hundredths place.

$r = \frac{9}{2} = 4.5$ $\quad r = \frac{7}{2} = 3.5$ $\quad \boxed{8\pi \text{ m}^2 \approx 25.13 \text{ m}^2}$

$A = \left(\frac{9}{2}\right)^2 \pi$ $\quad A = \left(\frac{7}{2}\right)^2 \pi$

$\frac{81}{4}\pi \text{ m}^2 - \frac{49}{4}\pi \text{ m}^2 = \frac{81-49}{4}\pi = \frac{32}{4}\pi = 8\pi$

B. Determine the area of the wrestling mat referred to as the "protection zone". Express your answer in exact form and as a decimal rounded to the nearest hundredths place.

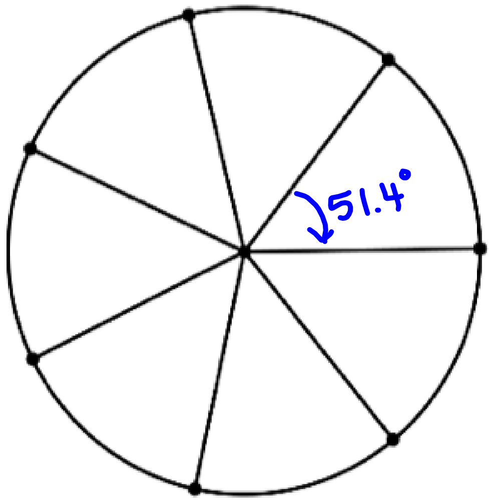
$\square = 12^2$

$= 144 \text{ m}^2 - \frac{81}{4}\pi \text{ m}^2$

$\boxed{144 - \frac{81}{4}\pi \text{ m}^2 \approx 80.38 \text{ m}^2}$

Notes:

Grandma wants to build her grandkids a Merry-Go-Round for her playground. This is not the mechanical type found at amusements parks, with wooden animals the kids ride; it is much smaller, where the kids sit on it and someone manually spins it around-and-around. Grandma has seven grandkids, so she wants her circular Merry-Go-Round to have seven sections, one for each grandkid, separated with a railing. She needs help figuring out its dimensions. A bird's eye view of the ride is shown below.



$360 \div 7 \approx 51.4° = \frac{360}{7}$

A. In order to help the contractors attach the rail that the kids will hold onto during their rides, determine the arc length between each rail if the ride is 10 ft in diameter. What would the arc length be if Grandma decides to go with a 12 ft diameter instead?

$d = 10 \text{ ft} \rightarrow C = 10\pi \text{ ft}$

$d = 12 \text{ ft} \rightarrow C = 12\pi \text{ ft}$

$\frac{\left(\frac{360}{7}\right)}{\frac{360}{1}} \cdot 10\pi$

$\frac{\cancel{360}}{7} \cdot \frac{1}{\cancel{360}} \cdot 10\pi$

$\frac{1}{7} \cdot 10\pi$

$\frac{10\pi}{7} \text{ ft}$

$\approx 4.49 \text{ ft}$

$\frac{1}{7} \cdot 12\pi$

$\frac{12\pi}{7}$

$\approx 5.39 \text{ ft}$

If the diameter is 10 ft, each arc length is 4.49 ft. If the diameter is 12 ft, each arc length is 5.39 ft.

B. Grandma is in a festive mood and wants to paint each section of the merry-go-round a different color, but she needs to figure out how much of each paint color she needs. To do this, she must find the area of each sector. What is the area of each sector for a diameter of 10 ft? Explain how you came up with your answer.

Each sector is $\frac{1}{7}$ of the entire circle.

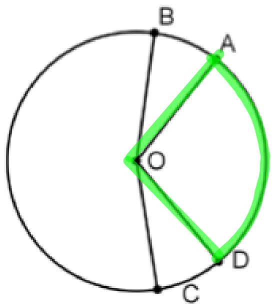
Area of a circle with 10 ft diameter is exactly $25\pi \text{ ft}^2$, 78.54 ft^2, so each of the 7 sectors has an area of 11.22 ft^2.

(erase to show)

A Sector of a Circle (#VOC)

The portion of a circle bounded by two radii and the intercepted arc.

Sector AOD



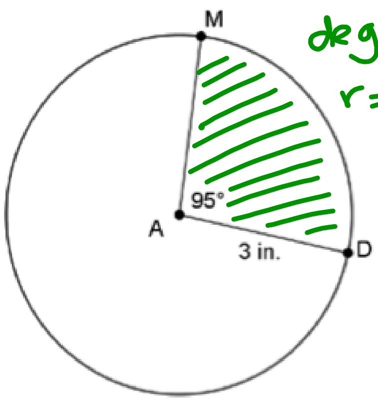
Area of a Sector of a Circle (#THM)

Given measure in degrees: $\frac{deg}{360} \cdot A$

Given measure in radians: $\frac{1}{2} \cdot \theta \cdot r^2$

Example 1: Find the area of each sector. Express your answer in exact form AND as a decimal rounded to the nearest tenths.

A.



deg = 95°

r = 3 in

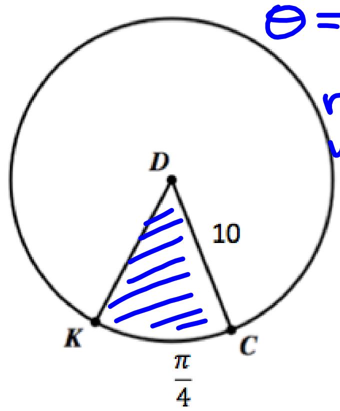
$\frac{deg}{360} \cdot A$

$\frac{95}{360} \cdot \pi (3)^2$

$\frac{19}{\cancel{72}_{8}} \cdot \frac{\cancel{9}^{1}\pi}{1}$

$\boxed{= \frac{19\pi}{8} \text{ in}^2 \quad \approx 7.5 \text{ in}^2}$

B.



$\theta = \frac{\pi}{4}$ rad

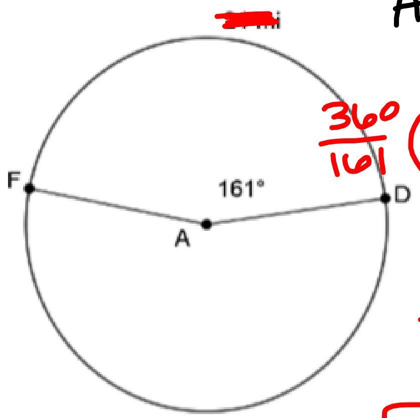
r = 10 units

$\frac{1}{2}\left(\frac{\pi}{4}\right)(10)^2$

$\frac{\pi}{\cancel{8}_{2}} \cdot \frac{\cancel{100}^{25}}{1}$

$\boxed{= \frac{25\pi}{2} \text{ units}^2 \quad \approx 39.3 \text{ units}^2}$

Example 2: In circle A (below) if the area of sector FAD is 140.5 in.^{2}, what is the length of the radius?



$$A \text{ of Sector} = \frac{deg}{360} \cdot \pi r^2$$

$$\frac{360}{161}(140.5) = \left(\frac{161}{360}\pi r^2\right) \cdot \frac{360}{161}$$

$$\frac{314.1615}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{r^2} = \sqrt{100}$$

$$r = 10$$

$\boxed{\text{The radius is 10 in.}}$

Example 3: Corey had a circular pizza with a diameter of 12 inches and then cut out a slice that had a central angle of 40°. What is the area of the sector that Corey's slice of pizza covered?

$deg = 40°$

$r = 6\text{ in}$

$$\frac{deg}{360} \cdot A$$

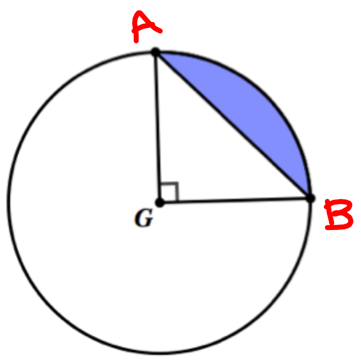
$$\frac{40}{360} \cdot \pi(6)^2$$

$$\frac{1}{9} \cdot \frac{36\pi}{1}$$

$$4\pi \text{ in}$$

$\boxed{\text{Corey's slice of pizza had an area of } 4\pi \text{ in}^2, 12.57 \text{ in}^2.}$

Example 4: Two radii of circle G form the legs of a right triangle. If the radius of circle G is 6 units, determine the area of shaded portion of the sector (i.e., the portion of the sector that is bounded by the chord and the circumference of the circle, as shown in the diagram below).



Area of Sector AGB − Area of △ AGB

deg = 90

r = 6

$\frac{90}{360} \cdot (6)^2 \pi$

$\frac{1}{4} \cdot 36\pi$

9π units^2

$\frac{1}{2}(6)(6)$

$\frac{1}{2}(36)$

18 units^2

$\boxed{\begin{array}{l} = 9\pi - 18 \text{ units}^2 \\ \approx 10.27 \text{ units}^2 \end{array}}$