Module 15d: Arc Length & Radians

Math Practice(s):

-Construct viable arguments & critique the reasoning of others.

-Look for & make use of structure.

Learning Target(s):

- Understand & apply the proportional relationship between an angle & its intercepted arc
- Explore, understand, & apply the relationship between arc angle measure & radian measure.

Homework:

HW#6: 15d #1-4

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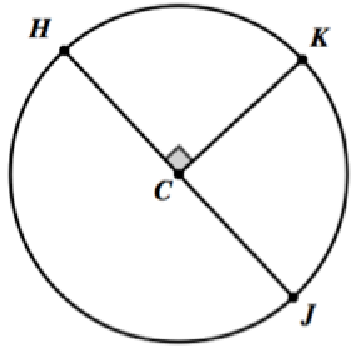
Warm-up

1. Circle C has a circumference of 40 feet and $\overline{CK} \perp \overline{HJ}$.



A. $m\angle HCK =$ 90 ° and $m\widehat{HK} =$ 90 °

B. $\widehat{HK}$ represents what fraction of the entire circle?

$\frac{90}{360} = \frac{1}{4}$

C. What is the arc LENGTH of $\widehat{HK}$ (measured in **feet**)?

$40 \div 4 \rightarrow \frac{1}{4}(40)$

10 ft

D. $\widehat{HJ}$ represents what fraction of the entire circle?

$\frac{180}{360} \rightarrow \frac{1}{2}$

E. What is the arc LENGTH of $\widehat{HJ}$ (measured in **feet**)?

$\frac{1}{2}(40) \rightarrow$ 20 ft

2. Circle B has a circumference of 12 meters, $m\angle LBC = 30°$ and $m\angle SBN = 120°$.

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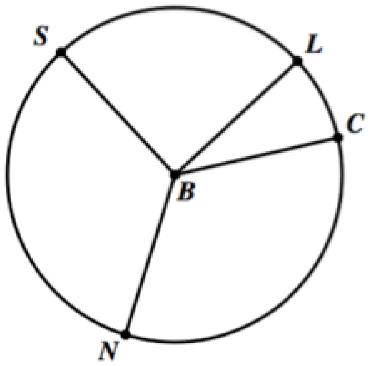
A. $m\widehat{LC} =$ 30 ° and $m\widehat{SN} =$ 120 °



B. $\widehat{LC}$ represents what fraction of the entire circle?

$\frac{30}{360} \rightarrow \frac{1}{12}$

C. What is the arc LENGTH of $\widehat{LC}$ (measured in **meters**)?

$\frac{1}{12}(12) \rightarrow$ 1 m

D. $\widehat{SN}$ represents what fraction of the entire circle?

$\frac{120}{360} \rightarrow \frac{1}{3}$

E. What is the arc LENGTH of $\widehat{SN}$ (measured in **meters**)?

$\frac{1}{3}(12) \rightarrow \frac{1}{3} \cdot \frac{12}{1} \rightarrow$ 4 m

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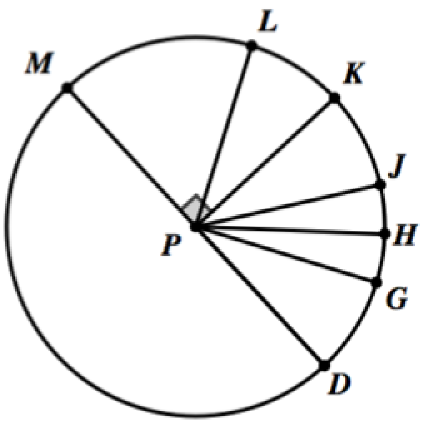
Circle P has a circumference of C units, $m\angle DPG = 30°$, $m\angle DPH = 45°$, $m\angle DPJ = 60°$, $m\angle DPK = 90°$, $m\angle DPL = 120°$, and ~~$m\angle DPG$~~ $m\angle DPM$ $= 180°$.



- **Length of** $\widehat{DM}$ = $\frac{1}{2}C$

  $\frac{180}{360} \rightarrow \frac{1}{2}$

- **Length of** $\widehat{MLK}$ = $\frac{1}{4}C$

  $\frac{90}{360} \rightarrow \frac{1}{4}$

- **Length of** $\widehat{DGH}$ = $\frac{1}{8}C$

  $\frac{45}{360} \rightarrow \frac{1}{8}$

- **Length of** $\widehat{DGJ}$ = $\frac{1}{6}C$

  $m\widehat{DGJ} = 60°$ $\frac{60}{360} \rightarrow \frac{1}{6}$

- **Length of** $\widehat{DJL}$ = $\frac{1}{3}C$

  $m\widehat{DJL} = 120°$ $\frac{120}{360} \rightarrow \frac{1}{3}$

- **Length of** $\widehat{DG}$ = $\frac{1}{12}C$

  $m\angle DPG = 30°$ $\frac{30}{360} \rightarrow \frac{1}{12}$

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The Length of an Arc on a Circle (#VOC)

The length of a circular arc is determined by the circles circumference and is the proportion of the circumference the arc represents.

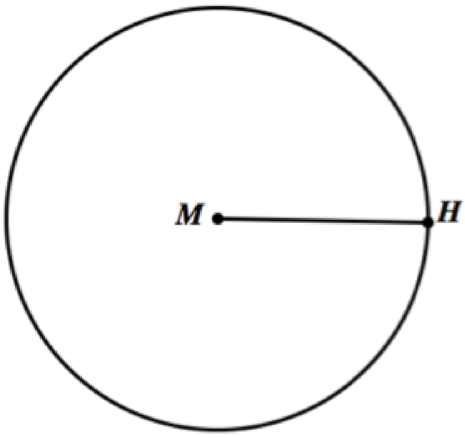
$\frac{deg}{360} \cdot C$

$C = 2\pi r$

$C = 2\pi(1)$

$C = 2\pi$ units

Example 1: Circle M has a radius of 1 unit.



A. Point V is somewhere on circle M such that $\widehat{HV}$ covers $\frac{1}{4}$ of the circle. What is the length of $\widehat{HV}$?

$\frac{1}{4}C \rightarrow \frac{1}{4}\frac{2\pi}{1} \rightarrow \frac{2\pi}{4}$

$\boxed{\frac{1}{2}\pi \text{ units}}$

B. Point W is somewhere on circle M such that $\widehat{HW}$ covers $\frac{1}{3}$ of the circle. What is the length of $\widehat{HW}$?

$\frac{2}{3}\pi$ units

C. Point X is somewhere on circle M such that $\widehat{HX}$ covers $\frac{1}{8}$ of the circle. What is the length of $\widehat{HX}$?

$\frac{1}{8} \cdot 2\pi$ $\boxed{\frac{1}{4}\pi \text{ units}}$

D. Point Y is somewhere on circle M such that $\widehat{HY}$ covers $\frac{1}{2}$ of the circle. What is the length of $\widehat{HY}$?

$\boxed{\pi \text{ units}}$

E. Point Z is somewhere on circle M such that $\widehat{HZ}$ covers $\frac{2}{3}$ of the circle. What is the length of $\widehat{HZ}$?

$\frac{2}{3} \cdot \frac{2\pi}{1}$ $\boxed{\frac{4}{3}\pi \text{ units}}$

F. Point A is somewhere on circle M such that $\widehat{HA}$ covers $\frac{1}{12}$ of the circle. What is the length of $\widehat{HA}$?

$\frac{1}{12} \cdot 2\pi$ $\boxed{\frac{1}{6}\pi \text{ units}}$

G. Point B is somewhere on circle M such that $\widehat{HB}$ covers $\frac{5}{8}$ of the circle. What is the length of $\widehat{HB}$?

$\frac{5}{8} \cdot \frac{2\pi}{1} \rightarrow \frac{10}{8}\pi$ $\boxed{\frac{5}{4}\pi \text{ units}}$

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Radian Measure (#VOC)

The ratio between the ___length___ of an arc intercepted by an angle and the ___radius___.

The most basic circle, with a radius of 1 unit, has a circumference of __2π__ units. So a circle, measured in degrees, has __$360°$__, can be measured in radians, having __2π__ radians.

This means… $360^\circ = 2\pi$ **radians.**

Dividing both sides by 2 gives… $180^\circ = \pi$.

Soooo… **1 radian** $= \frac{180}{\pi}$ **degrees** **&** **1 degree** $= \frac{\pi}{180}$ **radians**

Unit Circle

$150 \cdot \frac{\pi}{180}$

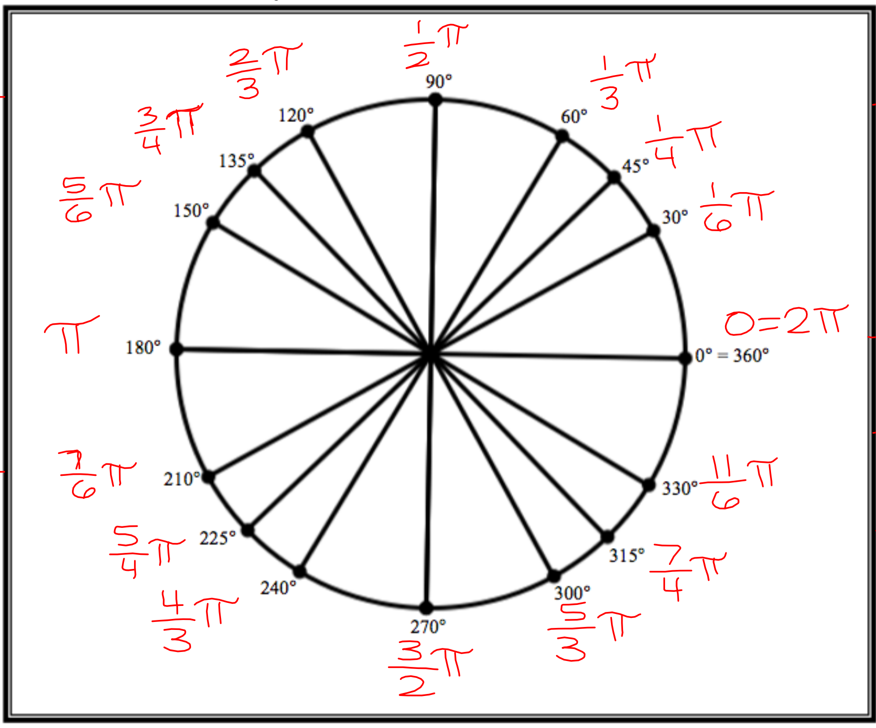
$210 \cdot \frac{\pi}{180}$

$225 \cdot \frac{\pi}{180}$

$300 \cdot \frac{\pi}{180}$

$\frac{30\pi}{18}$

$315 \cdot \frac{\pi}{180}$



$30 \cdot \frac{\pi}{180}$

$45 \cdot \frac{\pi}{180}$

$\frac{9}{1} \cdot \frac{\pi}{36}$

$60 \cdot \frac{\pi}{180}$

$120 \cdot \frac{\pi}{180}$

$135 \cdot \frac{\pi}{180}$

$\frac{27\pi}{36}$

Example 2: Convert the degrees to radians and radians to degrees. Be sure to indicate the units, either degrees or radians. You can use the abbreviation "rad" for radians.

A. $130^\circ = \frac{13}{18}\pi$ rad.

$130 \cdot \frac{\pi}{180}$

B. $\frac{\pi}{18} = 10^\circ$

$\frac{\cancel{\pi}}{18} \cdot \frac{180}{\cancel{\pi}} \rightarrow \frac{180}{18}$

C. $\frac{13\pi}{20} = 117^\circ$

$\frac{13\cancel{\pi}}{\cancel{20}} \cdot \frac{\cancel{180}\,9}{\cancel{\pi}}$

D. $240^\circ = \frac{4}{3}\pi$ rad.

E. $555^\circ = \frac{37}{12}\pi$ rad.

$555 \cdot \frac{\pi}{180} \rightarrow \frac{111}{36}\pi$

F. $\frac{16\pi}{3} = 960^\circ$

$\frac{16\pi}{3} \cdot \frac{180}{\pi}$

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Formula for determining the Length of an Arc (#THM)

The *Length of an Arc* (s) on a circle with radius, *r*, determined by angle *θ*, in radians, is given by

$$S = r \cdot \theta$$

Example 3: On a circle of radius 6 ft, what is the length of the arc between 2 points that are $\frac{\pi}{3}$ radians apart? Express your answer in exact form and as a decimal rounded to the nearest hundredths place.

$S = \frac{6}{1} \cdot \frac{\pi}{3} = \boxed{2\pi \text{ ft} \approx 6.28 \text{ ft}}$

Example 4: The clock faces of Big Ben in London measure 23 ft in diameter. If a fly was sitting at the tip of the minute hand, how far would it travel in 5 minutes. Express your answer in radians in exact form and as a decimal rounded to the nearest hundredths place.

$\frac{5 \text{ min}}{60 \text{ min}} \rightarrow \frac{1}{12}$ of clock

$\frac{1}{12}$ of clock $\cdot 360^\circ = \frac{360}{12} = 30^\circ$

$30^\circ \cdot \frac{\pi}{180} \rightarrow \frac{30}{180}\pi \rightarrow \frac{3}{18}\pi \rightarrow \frac{1}{6}\pi$ radians

$S = r \cdot \theta$

$= \frac{23}{2} \cdot \frac{1}{6}\pi$

$$\boxed{\begin{aligned} &= \frac{23}{12}\pi \text{ ft} \\ &\approx 6.02 \text{ ft} \end{aligned}}$$