Module 15c: Inscribed & Circumscribed Angles

Math Practice(s):

   -Construct viable arguments & critique the reasoning of others.

   -Look for & make use of structure.

Learning Target(s):

   - Discover & apply the relationship between
      + an inscribed angle & its intercepted arc
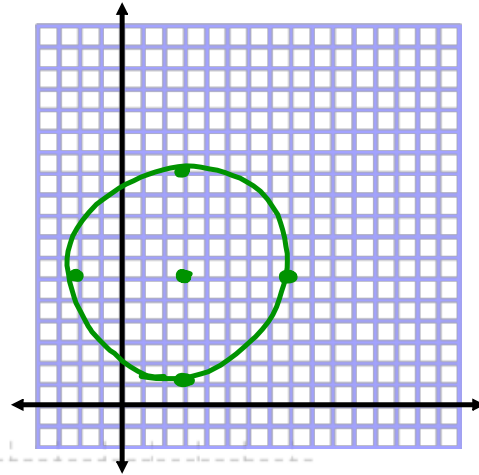   + a circumscribed angle & its intercepted arc

Homework:

   HW#5: 15c #1-6

Warm-up

1. Sketch below the circle defined by $(x - 3)^2 + (y - 6)^2 = 25$, and label its center point C.

center: $(3, 6)$

radius: $\sqrt{25} = 5$ units

2. Using the equation for the circle you graphed above to verify that the point (8, 6) lies on the circle.

$$(x - 3)^2 + (y - 6)^2 = 25$$

$$(8 - 3)^2 + (6 - 6)^2 = 25$$

$$(5)^2 + (0)^2 = 25$$

$$25 = 25 \checkmark$$

x y

3. Using the equation for the circle you graphed above to verify that the point (7, 9) lies on the circle.

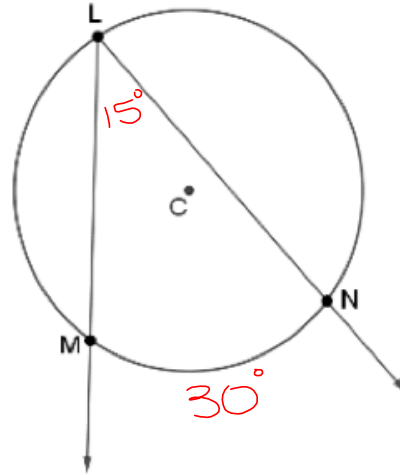$$(7 - 3)^2 + (9 - 6)^2 = 25$$

$$(4)^2 + (3)^2 = 25$$

$$16 + 9 = 25$$

$$25 = 25 \checkmark$$

x y

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2

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Inscribed Angle (#VOC)

An angle whose ___vertex___ is

___on___ the circle and whose rays contain

___chords___ of the circle.

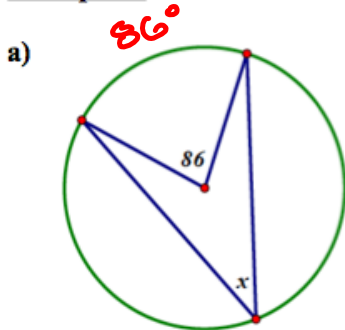Inscribed angle, ∠MLN, intercepts arc, $\overparen{MN}$.

The Inscribed Angle Theorem (#THM)

The measure of the inscribed angle is

___half___ the measure of the

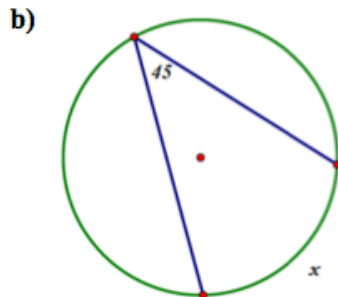___intercepted___ ___arc___.

$$m\angle MLN = \tfrac{1}{2} \, m\overparen{MN}$$

OR

$$m\overparen{MN} = 2 \cdot m\angle MLN$$

(Circle diagram with points L, M, N, center C; angle $15°$ at L and arc $30°$)

Example 1: Find the value of the variable.

a)

$86°$

86

x

$$x = \tfrac{1}{2}(86)$$

$$\boxed{x = 43°}$$

b)

45

x

$$x = 2(45)$$

$$\boxed{x = 90°}$$

c)

$9x + 8$

$5x$
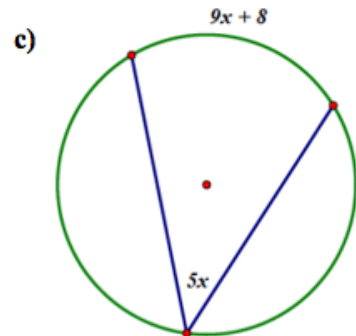
$$2 \cdot (5x) = 9x + 8$$

OR

$$5x = \tfrac{1}{2}(9x + 8)$$

$$\rightarrow 10x = 9x + 8$$

$$\boxed{x = 8}$$
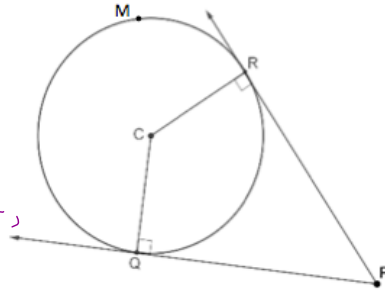
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3

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Circumscribed Angle (#VOC)

An angle whose vertex is ___outside___

the circle and whose rays are

___tangent___ to the circle.
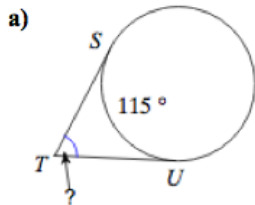
Circumscribed angle, ∠QPR, intercepts arc, $\overparen{QR}$.

The Circumscribed Angle Theorem (#THM)

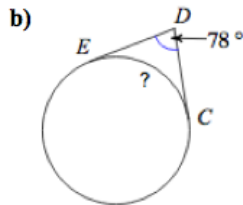The measure of the circumscribed angle is ___supplementary___

to the measure of the ___intercepted___ ___arc___.
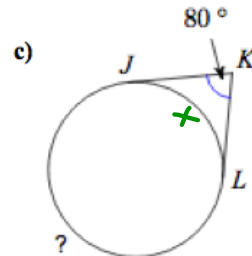
$m\angle RPQ + m\overparen{QR} = 180$

Example 2: Find the value of the variable.

a)

115°

?

$? + 115 = 180$

$? = 180 - 115$

$\boxed{? = 65°}$

b)

$-78°$

?

$? + 78 = 180$

$? = 180 - 78$

$\boxed{? = 102°}$

c)

80°

x

?

$x + 80 = 180$

$x = 100$

$? + x = 360$

$? + 100 = 360$

$\boxed{? = 260°}$

d)

78°

L   K

a

M
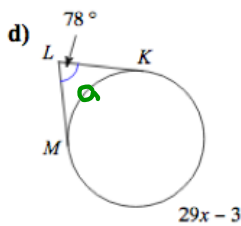
$29x - 3$

$78 + a = 180$

$a = 180 - 78$

$\underline{a = 102}$

$102 + 29x - 3 = 360$

$29x + 99 = 360$

$29x = 261$

$\boxed{x = 9}$

e)

C

118°

A $21x - 1$ B
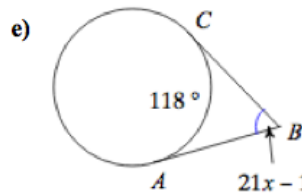
$118 + 21x - 1 = 180$

$21x + 117 = 180$

$21x = 63$

$\boxed{x = 3}$

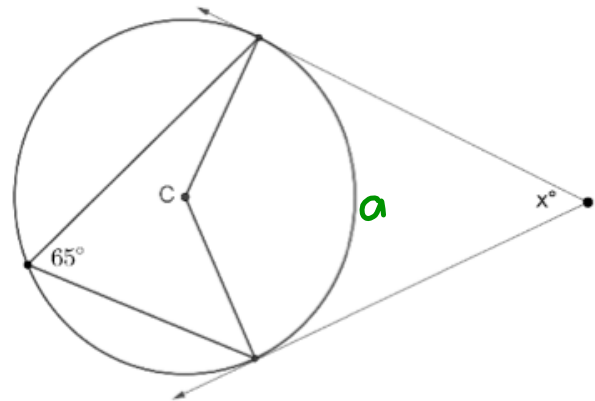**Example 3:** Find the value of the variable.

$a = 2(65)$

$a = 130°$

$x + a = 180$

$x + 130 = 180$

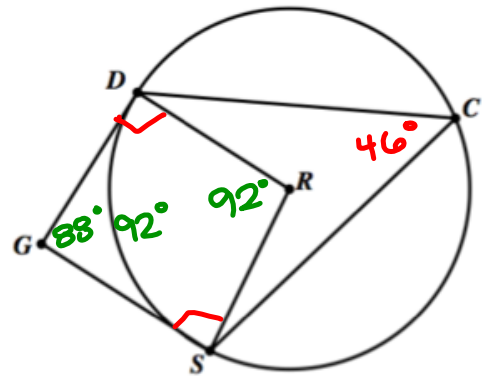$\boxed{x = 50}$

Example 4:

$\overline{GD}$ and $\overline{GS}$ are tangent to circle R
$m\angle DCS = 46°$

$m\overparen{DS} = 2(46)$
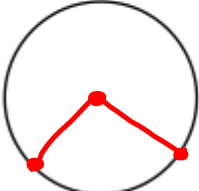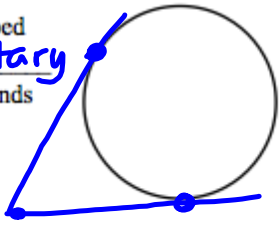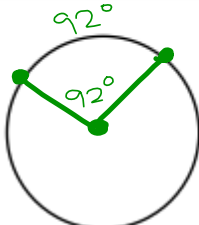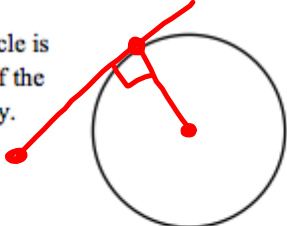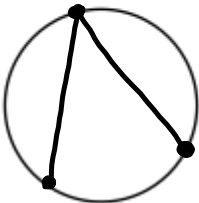
$\boxed{m\overparen{DS} = 92°}$ $\boxed{m\angle DRS = 92°}$

$m\angle DGS + 92° = 180$

$\boxed{m\angle DGS = 88°}$

Match the definition to the word, then draw in an appropriate diagram.

Vocabulary	Diagram	Definition
Central Angle • The measure of a central angle is ___equal to___ the measure of the arc it subtends. • Definition: ___E___		A. An angle whose vertex is on the circle and whose rays contain chords of the circle
Circumscribed Angle • The measure of a circumscribed angle is ___supplementary___ to the central angle that subtends the same arc. • Definition: ___D___		B. The measure of the central angle that determines the arc.
Arc Angle Measure • Arc angle measure is determined by the proportion of the circumference that the arc represents. • Definition: ___B___	92° 92°	C. A line that lies in the plane of the circle and intersects the circle at exactly one point.
Tangent Line • A line that is tangent to a circle is perpendicular to the radius of the circle at the point of tangency. • Definition: ___C___		D. An angle whose rays are tangent to the circle.
Inscribed Angle • The measure of an inscribed angle is equal to one-half the measure of the arc that it subtends. • Definition: ___A___		E. An angle whose vertex is the center of the circle and whose sides are radii.

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