

## Rational Functions 1: Multiplying & Dividing

Standards:A-APR.7  
A-SSE.1a

#11 HW:  
Rats 1 #1-8

### Learning Target:

When are you allowed to cancel in a fraction?

How do you multiply & divide rational functions?

(erase to show)

**Definition:** A **Rational Function** is a function that can be written as the QUOTIENT of two polynomials

$$f(x) = \frac{g(x)}{h(x)}$$

where  $h$  is not the zero polynomial (i.e.  $h(x) \neq 0$  for all  $x$ ).

**Example:** The function defined as  $f(x) = \frac{x^2 - 1}{x + 2}$  is a rational function.

A rational function is in simplified form if its numerator & denominator have no common factors (other than 1).

i.e.  $\frac{2}{3}$

## Simplifying Rational Expressions

Let a, b, c be non-zero real #s.

(erase to show)

$$\frac{\cancel{ac}}{\cancel{bc}} = \frac{a}{b}$$

(cancel common factors)

$$\text{ex. } \frac{6}{9} = \frac{2 \cdot \cancel{3}}{3 \cdot \cancel{3}} = \frac{2}{3}$$

**Usually has 3 steps:**

- 1. Factor numerator & denominator.**
- 2. Divide out (cancel) any common factors.**
- 3. Simplify if possible.**

Simplify the rational functions, and state the excluded values (i.e. values of x for which the expression is not defined).

Operation:                      Rational Expression:                      Excluded Value:

Reduce

$$\frac{16x^2}{28x}$$

$$\frac{4 \cdot 4 \cdot x \cdot x}{7 \cdot 4 \cdot x}$$

$$\frac{4x}{7}$$

$$\frac{28x \neq 0}{28 \quad 28}$$

$$x \neq 0$$

$$\frac{2x-6}{x-3}$$

$$\frac{2(x-3)}{(x-3)}$$

$$2$$

$$x-3 \neq 0$$

$$x \neq 3$$

$$\frac{4x-12}{2x^2-2x-12}$$

$$\frac{4(x-3)}{2(x^2-x-6)}$$

$$\frac{2 \cancel{4}(x-3)}{\cancel{2}(x+2)\cancel{(x-3)}}$$

$$\frac{2}{(x+2)}$$

$$2x^2-2x-12 \neq 0$$

$$2(x+2)(x-3) \neq 0$$

$$2 \neq 0 \quad x+2 \neq 0$$

$$x-3 \neq 0$$

$$x \neq -2, 3$$

Simplify:

$$\frac{x^2 - 5x - 6}{x^2 - 1}$$

$$\frac{(x-6)(\cancel{x+1})}{(\cancel{x+1})(x-1)}$$

$$\frac{(x-6)}{(x-1)}$$

$$\boxed{\frac{(x-6)}{(x-1)}}$$

excl. value(s):

$$x^2 - 1 \neq 0$$

$$(x+1)(x-1) \neq 0$$

$$x+1 \neq 0 \quad x-1 \neq 0$$

$$x \neq -1 \quad x \neq 1$$

$$\boxed{x \neq -1, 1}$$

When **multiplying** the same rules apply as multiplying fractions. **Multiply across** . (Simplify if possible)

(erase to show)

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Operation:Rational Expression:Excluded Value:

Multiply

$$\frac{5x^4}{2x} \cdot \frac{8}{x^2} = \frac{40x^4}{2x^3}$$

$$2x \neq 0 \quad x^2 \neq 0$$

$$\boxed{x \neq 0}$$

$$\frac{\cancel{2} \cdot \cancel{20} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{2} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}$$

$$\boxed{20x}$$

$$\frac{5x^2y}{2xy^3} \cdot \frac{6x^3y^2}{10y}$$

$$2xy^3 \neq 0 \quad 10y \neq 0$$

$$\frac{30x^5y^3}{20xy^4}$$

$$\boxed{x \neq 0, y \neq 0}$$

$$\frac{30}{20} \cdot \frac{x^5}{x} \cdot \frac{y^3}{y^4}$$

$$\frac{3}{2} \cdot x^{5-1} \cdot y^{3-4}$$

$$\frac{3}{2} \cdot x^4 \cdot y^{-1}$$

$$\boxed{\frac{3x^4}{2y}}$$

Operation:                      Rational Expression:                      Excluded Value:

Multiply

$$\frac{x+1}{2} \cdot \frac{x-1}{x^2} =$$

$$\cancel{2} \neq 0 \quad x^2 \neq 0$$

$$\boxed{x \neq 0}$$

$$\boxed{\frac{(x+1)(x-1)}{2x^2}}$$

$$\frac{5x+15}{2x} \cdot \frac{8x-2}{x^2-9}$$

$$2x \neq 0 \quad x^2 - 9 \neq 0$$

$$2x \neq 0 \quad (x+3)(x-3) \neq 0$$

$$\frac{\cancel{5}(x+3)}{\cancel{2}x} \cdot \frac{\cancel{2}(4x-1)}{\cancel{(x+3)}(x-3)}$$

$$\boxed{x \neq 0} \quad x+3 \neq 0 \quad x-3 \neq 0$$

$$\boxed{x \neq -3, 3}$$

$$\boxed{\frac{5(4x-1)}{x(x-3)}}$$

$$\boxed{x \neq 0, -3, 3}$$

$$\frac{3x-27x^3}{3x^2-2x-1} \cdot \frac{3x^2-4x+1}{3x}$$

$$3x^2-2x-1 \neq 0 \quad 3x \neq 0$$

$$(3x+1)(x-1) \neq 0 \quad 3x \neq 0$$

$$3x+1 \neq 0 \quad x-1 \neq 0 \quad 3x \neq 0$$

$$3x \neq -1 \quad x \neq 1 \quad x \neq 0$$

$$x \neq -\frac{1}{3}$$

$$\frac{\cancel{3}x(1-9x^2)}{\cancel{(3x^2-2x-1)}_{3x+1}} \cdot \frac{(3x^2-4x+1)}{\cancel{3}x}$$

$$\frac{\cancel{(1+3x)}(1-3x)}{\cancel{(3x+1)}(x-1)} \cdot \frac{(3x-1)\cancel{(x+1)}}{1}$$

$$\boxed{(1-3x)(3x-1)}$$

$$\boxed{x \neq -\frac{1}{3}, 1, 0}$$

**Division:**

Remember that dividing by a fraction is the same as  
**multiplying by the reciprocal** (Copy-Dot-Flip)

(erase to show)

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

**Operation:****Rational Expression:****Excluded Value:**

Divide

$$\frac{x^2 - 1}{3x} \div \frac{x+1}{x-9}$$

$$3x \neq 0 \quad x-9 \neq 0$$

$$\boxed{x \neq 0} \quad \boxed{x \neq 9}$$

$$\frac{(x+1)(x-1)}{3x} \div \frac{(x+1)}{(x-9)}$$

$$x+1 \neq 0$$

$$\boxed{x \neq -1}$$

$$\frac{\cancel{(x+1)}(x-1)}{3x} \cdot \frac{(x-9)}{\cancel{(x+1)}}$$

$$\boxed{x \neq 0, 9, -1}$$

$$\boxed{\frac{(x-1)(x-9)}{3x}}$$



Operation:                      Rational Expression:                      Excluded Value:

Divide                       $\frac{3}{4x-8} \div \frac{x^2+3x}{x^2+x-6}$                        $4(x-2) \neq 0$      $(x-2)(x+3) \neq 0$   
 ~~$4 \neq 0$~~      $x-2 \neq 0$      $x-2 \neq 0$   
 $x+3 \neq 0$

$$\frac{3}{4(x-2)} \div \frac{x(x+3)}{(x-2)(x+3)}$$

$$\boxed{x \neq 2} \quad \boxed{x \neq 2 \quad x \neq -3}$$

$$\frac{3}{4(x-2)} \cdot \frac{\cancel{(x-2)}\cancel{(x+3)}}{x\cancel{(x+3)}}$$

$$x(x+3) \neq 0$$

$$x \neq 0 \quad x+3 \neq 0$$

$$\boxed{x \neq 0 \quad x \neq -3}$$

$$\boxed{\frac{3}{4x}}$$

$$\boxed{x \neq 2, 0, -3}$$

$$\frac{6x^2+7x-3}{x-1} \div (3x-1)$$

$$x-1 \neq 0 \quad 3x-1 \neq 0$$

$$x \neq 1 \quad 3x \neq 1$$

$$x \neq \frac{1}{3}$$

$$\frac{(2x+3)(3x-1)}{(x-1)} \div \frac{(3x-1)}{1}$$

$$\boxed{x \neq 1, \frac{1}{3}}$$

$$\frac{(2x+3)\cancel{(3x-1)}}{(x-1)} \cdot \frac{1}{\cancel{(3x-1)}}$$

$$\boxed{\frac{(2x+3)}{(x-1)}}$$