

## Module 14b: The Equation of a Circle

### **Math Practice(s):**

- Model with mathematics.
- Look for & make use of structure.

### **Learning Target(s):**

- Using coordinate geometry, derive the equation of a circle with a given center and radius.
- Given the equation of a circle, verify whether or not a given point lies on the circle.

### **Homework:**

HW#2: 14b #1-3

**Warm-up**

1. Does the point  $(3, 5)$  lie on the graph of  $y = 2x + 1$ ? Justify your answer by substituting the coordinates into the equation to determine if it will lie on the graph.

$$\begin{aligned} (5) &= 2(3) + 1 \\ 5 &= 6 + 1 \\ 5 &= 7 \end{aligned}$$

No,  $(3, 5)$  does not lie on the line  $y = 2x + 1$ .

2. Does the point  $(-2, 3)$  lie on the graph of  $y = x^2 + 7$ ? Justify your answer by substituting the coordinates into the equation to determine if it will lie on the graph.

$$\begin{aligned} (3) &= (-2)^2 + 7 \\ 3 &= (4) + 7 \\ 3 &= 11 \end{aligned}$$

No,  $(-2, 3)$  does not lie on the parabola  $y = x^2 + 7$ .

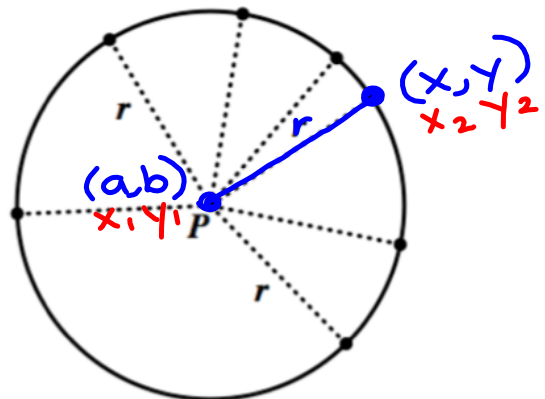
A circle centered at the point P with radius  $r$  is the set of all points a distance  $r$  from P.

center  $(a, b)$   
radius:  $r$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

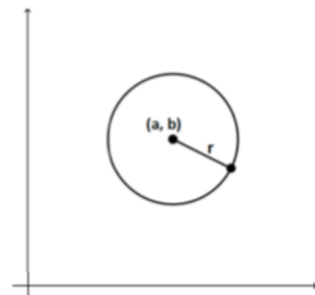
$$r^2 = \sqrt{(x - a)^2 + (y - b)^2}$$

$$r^2 = (x - a)^2 + (y - b)^2$$



The circle in a Cartesian Plane centered at  $(a, b)$  with radius  $r$  is the set of all points satisfying the equation

$$(x - a)^2 + (y - b)^2 = r^2$$



**Example 1:**

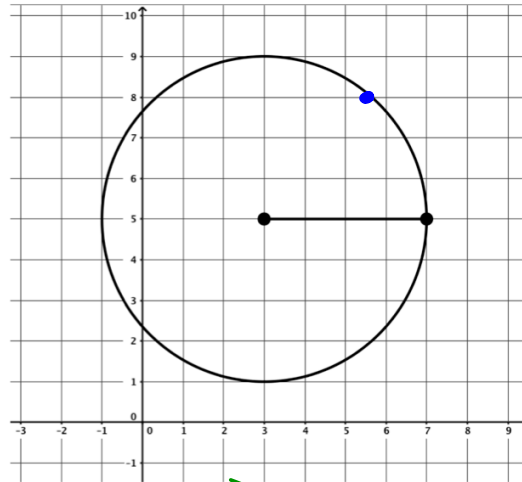
$$(x-a)^2 + (y-b)^2 = r^2$$

- A. Determine the equation for the circle shown in the coordinate plane below.
- B. Then, identify four points that lie on the circle. Verify your answers by showing the coordinates satisfy the equation you created.
- C. Does the point (5.5, 8) lie on the circle? Justify why or why not.

A) center:  $(3, 5)$   
 radius: 4 units

$$(x-(3))^2 + (y-(5))^2 = (4)^2$$

$$(x-3)^2 + (y-5)^2 = 16$$



B)  $(3, 9)$

$$(3-3)^2 + (9-5)^2 = 16$$

$$(0)^2 + (4)^2 = 16$$

$$0 + 16 = 16$$

$$16 = 16$$

✓

$$((7)-3)^2 + ((5)-5)^2 = 16$$

$$(7, 5) \quad (4)^2 + (0)^2 = 16$$

$$16 + 0 = 16$$

$$16 = 16$$

✓

$(3, 1)$

$$((3)-3)^2 + ((1)-5)^2 = 16$$

$$(0)^2 + (4)^2 = 16$$

$$0 + 16 = 16$$

$$16 = 16$$

✓

$(-1, 5)$

$$((-1)-3)^2 + ((5)-5)^2 = 16$$

$$(-4)^2 + (0)^2 = 16$$

$$16 + 0 = 16$$

$$16 = 16$$

✓

C)  $(5.5, 8)$

x y

$$((5.5)-3)^2 + ((8)-5)^2 = 16$$

$$(2.5)^2 + (3)^2 = 16$$

$$6.25 + 9 = 16$$

$$15.25 = 16$$

•  $(5.5, 8)$  is not on the circle.

**Example 2:**

- A. Determine the equation of the circle shown in the coordinate plane below.
- B. Then, identify three points that lie on the circle. Verify your answers by showing the coordinates satisfy the equation you created.

A) center :  $(1, 2)$   
 radius : 3 units  
 $(x - (1))^2 + (y - (2))^2 = (3)^2$

$$(x - 1)^2 + (y - 2)^2 = 9$$

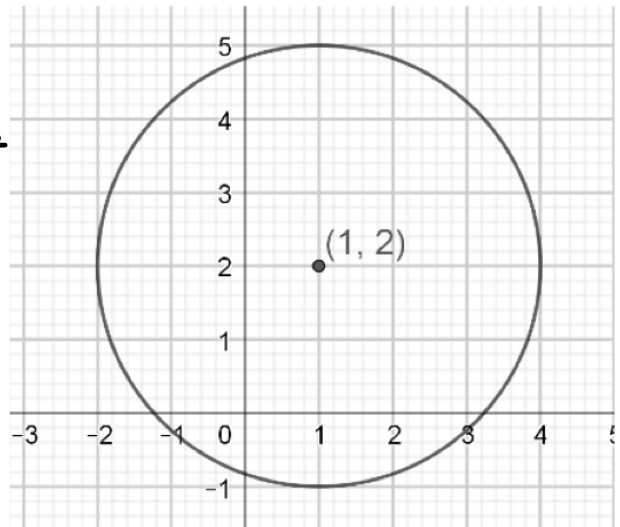
B)  $(4, 2)$   
~~x~~ ~~y~~

$$((4) - 1)^2 + ((2) - 2)^2 = 9$$

$$(3)^2 + (0)^2 = 9$$

$$9 + 0 = 9$$

$$9 = 9$$



$(1, 5)$   $(-2, 2)$

**Example 3:**

- A. Determine the equation of the circle with center  $(-1, 3)$  that contains the point  $(-4, -1)$ .
- B. Then, identify two additional points that lie on the circle. Verify your answers by showing the coordinates satisfy the equation you created.

A) center :  $(-1, 3)$   
 radius : 5 units

$$(x - (-1))^2 + (y - (3))^2 = (5)^2$$

$$(x + 1)^2 + (y - 3)^2 = 25$$

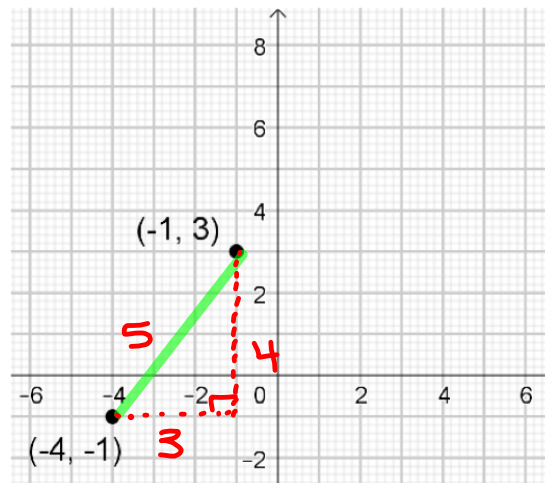
B)  $(4, 3)$

$$((4) + 1)^2 + ((3) - 3)^2 = 25$$

$$(5)^2 + (0)^2 = 25$$

$$25 = 25$$

✓



$(-1, -2)$

$$((-1) + 1)^2 + ((-2) - 3)^2 = 25$$

$$(0)^2 + (-5)^2 = 25$$

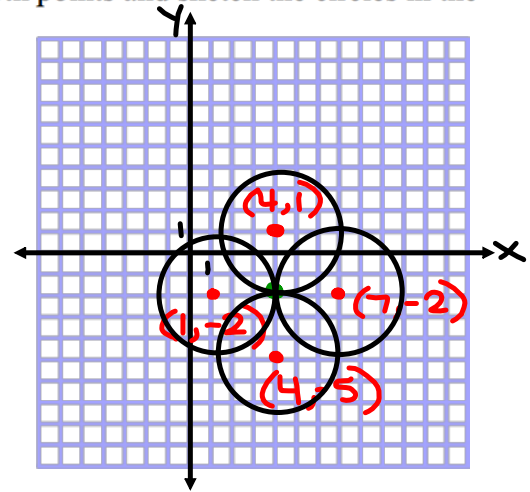
$$25 = 25$$

✓

**Example 4:**

The point  $(4, -2)$  lies on circle  $G$ . If the radius of circle  $G$  is 3 units, identify three points that could possibly be the center of circle  $G$ . State the coordinates of both points and sketch the circles in the coordinate plane below.

$(4, 1), (7, -2), (4, -5), (1, -2)$



**Example 5:**

$$(x-a)^2 + (y-b)^2 = r^2$$

Answer the following questions regarding the circle  $C$  defined by  $x^2 + (y - 3)^2 = 16$ .

A. The center of  $C$  is located at the point  $P = ( \underline{0} , \underline{3} )$

B. The radius of circle  $C$  is  $r = \underline{4 \text{ units}}$   $\sqrt{r^2} = \sqrt{16}$

C. Circle each of the following points that lie on circle  $C$ :

$$(0)^2 + ((-1) - 3)^2 = 16$$

$(0, -1)$

~~$(0, 1)$~~

$(-4, 3)$

$(0, 7)$

$$(0)^2 + (-4)^2 = 16$$

$$16 = 16$$

✓

$$(0)^2 + ((1) - 3)^2 = 16$$

$$(0)^2 + (-2)^2 = 16$$

$$4 = 16$$

✗

$$(-4)^2 + ((3) - 3)^2 = 16$$

$$(-4)^2 + (0)^2 = 16$$

$$16 = 16$$

✓

$$(0)^2 + ((7) - 3)^2 = 16$$

$$(0)^2 + (4)^2 = 16$$

$$16 = 16$$

✓