Radicals 4b – Simplifying with Rational Exponents

Standards: N-RN.1 & N-RN.2

#11HW:
Rads 4b
#1-17

Learning Target:
How do you simplify a radical expression with variables in it?

In the previous lesson we reviewed our rules of exponents. We can also apply these rules to rational exponents.

If $m = \frac{1}{n}$ then we can rewrite the property in radical notation:

Power of a Product: $(ab)^m = a^m b^m$

$$\boxed{\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}}$$

Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$$\boxed{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$$

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Ex#1: Simplify using exponent rules. Write your answer using a radical, if needed:

a) $6^{1/2} \cdot 6^{1/3}$

$6^{\frac{3}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{2}{2}}$

$6^{\frac{3}{6}+\frac{2}{6}}$

$6^{\frac{3+2}{6}}$

$6^{5/6}$

$\boxed{(\sqrt[6]{6})^5} \rightarrow \sqrt[6]{6^5}$

b) $(27^{1/3} \cdot 6^{1/4})^2$

$(27^{1/3})^{\frac{2}{1}} \cdot (6^{1/4})^{\frac{2}{1}}$

$27^{2/3} \cdot 6^{1/2}$

$(\sqrt[3]{27})^2 \cdot (\sqrt[2]{6})^1$

$(3)^2 \cdot \sqrt{6}$

$9 \cdot \sqrt{6}$

$\boxed{9\sqrt{6}}$

c) $(4^3 \cdot 2^3)^{-1/3}$

$\frac{1}{(4^3 \cdot 2^3)^{1/3}}$

$\frac{1}{(4^3)^{1/3} \cdot (2^3)^{1/3}}$

$\frac{1}{4 \cdot 2}$

$\frac{1}{8}$

d) $\frac{6^1}{6^{3/4}}$

$6^{1-\frac{3}{4}}$

$6^{1/4}$

$\boxed{\sqrt[4]{6}}$

6

2 3

e) $\left(\frac{18^{1/4}}{9^{1/4}}\right)^3$

$\frac{(18^{1/4})^3}{(9^{1/4})^3}$

$\frac{18^{3/4}}{9^{3/4}}$

$\left(\frac{18}{9}\right)^{3/4}$

$2^{3/4}$

$\boxed{(\sqrt[4]{2})^3}$ $\sqrt[4]{2^3}$

$\sqrt[4]{8}$

f) $\frac{\sqrt{10}}{\sqrt[6]{10}}$ $\frac{10^{1/2}}{10^{1/6}}$

$10^{\frac{3}{3}\cdot\frac{1}{2}-\frac{1}{6}}$

$10^{\frac{3}{6}-\frac{1}{6}}$

$10^{\frac{3-1}{6}}$

$10^{1/3}$

$\boxed{\sqrt[3]{10}}$

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Ex#2: Simplify using radical properties

a) $\sqrt[3]{25} \cdot \sqrt[3]{5}$

$\sqrt[3]{25 \cdot 5}$

$\sqrt[3]{125}$

5

b) $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}}$

$\sqrt[3]{\dfrac{32}{4}}$

$\sqrt[3]{8}$

2

c) $\dfrac{\sqrt[4]{162}}{\sqrt[4]{2}}$

$\sqrt[4]{\dfrac{162}{2}}$

$\sqrt[4]{81}$

3

81
9 9
3 3 3 3

Simplest Form: apply properties of radicals & remove any perfect nth powers.
Leave in Radical Form.

Ex#3: Write in Simplest Form. (No Decimals!)

a) $\sqrt[4]{64}$

$\sqrt[4]{2^4 \cdot 2^2}$

$\cancel{\sqrt[4]{2^4}} \cdot \sqrt[4]{2^2}$

$2 \cdot \sqrt[4]{4}$

$\boxed{2\sqrt[4]{4}}$

64
8 8
2 4 2 4
2 2 2 2

b) $\sqrt[3]{270}$

$\sqrt[3]{3^3 \cdot 2 \cdot 5}$

$\cancel{\sqrt[3]{3^3}} \cdot \sqrt[3]{10}$

$\boxed{3\sqrt[3]{10}}$

270
27 10
9 3 2 5
3 3

We can also apply the properties of rational exponents & radicals to expressions with variables.

$$\sqrt[n]{x^n} = x \qquad \text{if n is odd}$$

$$\sqrt[n]{x^n} = |x| \qquad \text{if n is even}$$

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<u>Ex#4</u>: Simplify. Write your answer in radical form. Assume all variables are positive.

a) $\sqrt[3]{27z^9}$

27
9 3
3 3

$\cancel{\sqrt[3]{3^3}} \cdot \sqrt[3]{z^9}$

$3 \cdot \cancel{\sqrt[3]{z^3}} \cdot \cancel{\sqrt[3]{z^3}} \cdot \cancel{\sqrt[3]{z^3}}$

$\boxed{3z^3}$

b) $\sqrt[4]{12d^4e^9f^{14}}$

12
2 6
2 3

$\sqrt[4]{12^1} \cdot \cancel{\sqrt[4]{d^4}} \cdot \sqrt[4]{e^{9}} \cdot \sqrt[4]{f^{14}}$

$\sqrt[4]{12} \cdot d \cdot \cancel{\sqrt[4]{e^4}} \cdot \cancel{\sqrt[4]{e^4}} \cdot \sqrt[4]{e^1} \cdot \cancel{\sqrt[4]{f^4}} \cdot \cancel{\sqrt[4]{f^4}} \cdot \cancel{\sqrt[4]{f^4}} \cdot \sqrt[4]{f^2}$

$\sqrt[4]{12} \cdot d \cdot e^2 \cdot \sqrt[4]{e} \cdot f^3 \cdot \sqrt[4]{f^2}$

$\boxed{de^2f^3\sqrt[4]{12ef^2}}$

c) $\sqrt[4]{320x^8y^{11}z^6}$

320
32 10
2 16 2 5
2 8
2 4
2 2

$\sqrt[4]{320} \cdot \sqrt[4]{x^8} \cdot \sqrt[4]{y^{11}} \cdot \sqrt[4]{z^6}$

$\cancel{\sqrt[4]{2^4}} \cdot \sqrt[4]{2\cdot2\cdot5} \cdot \cancel{\sqrt[4]{x^4}} \cdot \cancel{\sqrt[4]{x^4}} \cdot \cancel{\sqrt[4]{y^4}} \cdot \cancel{\sqrt[4]{y^4}} \cdot \sqrt[4]{y^3} \cdot \cancel{\sqrt[4]{z^4}} \cdot \sqrt[4]{z^2}$

$2 \cdot \sqrt[4]{20} \cdot x^2 \cdot y^2 \cdot \sqrt[4]{y^3} \cdot z \cdot \sqrt[4]{z^2}$

$\boxed{2x^2y^2z\sqrt[4]{20y^3z^2}}$

d) $\sqrt[5]{\dfrac{x^5}{y^{10}}}$

$\dfrac{\cancel{\sqrt[5]{x^5}}}{\sqrt[5]{y^{10}}} \qquad \dfrac{x}{\cancel{\sqrt[5]{y^5}} \cdot \cancel{\sqrt[5]{y^5}}} \qquad \dfrac{x}{y^2}$

e) $\sqrt[3]{8r^3s^5t^{10}}$

$\sqrt[3]{8} \cdot \cancel{\sqrt[3]{r^3}} \cdot \sqrt[3]{s^5} \cdot \sqrt[3]{t^{10}}$

$2 \cdot r \cdot \cancel{\sqrt[3]{s^3}} \cdot \sqrt[3]{s^2} \cdot \cancel{\sqrt[3]{t^3}} \cdot \cancel{\sqrt[3]{t^3}} \cdot \cancel{\sqrt[3]{t^3}} \cdot \sqrt[3]{t^1}$

$\boxed{2rst^3\sqrt[3]{s^2t}}$

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Ex#5: Simplify. Assume all variables are positive. Write your answer in exponential form using only positive exponents.

a) $(16g^4h^2)^{1/2}$

① $\sqrt{16g^4h^2}$

$\sqrt{16}\cdot\sqrt{g^4}\cdot\sqrt{h^2}$

$4\cdot\sqrt{g^2}\cdot\sqrt{g^2}\cdot h$

$\boxed{4g^2h}$

② $(16)^{1/2}\cdot(g^4)^{1/2}\cdot(h^2)^{1/2}$

$\sqrt{16}\cdot g^2\cdot h$

$\boxed{4g^2h}$

b) $\dfrac{18rs^{2/3}}{6r^{1/4}t^{-3}}$

$\frac{18}{6}\cdot\frac{r}{r^{1/4}}\cdot\frac{s^{2/3}}{1}\cdot\frac{1}{t^{-3}}$

$3\cdot r^{1-1/4}\cdot s^{2/3}\cdot t^3$

$\boxed{3r^{3/4}s^{2/3}t^3}$

c) $(625j^8k^4)^{1/4}$

① $\sqrt[4]{625j^8k^4}$

$\sqrt[4]{625}\cdot\sqrt[4]{j^8}\cdot\sqrt[4]{k^4}$

$\sqrt[4]{5^4}\cdot\sqrt[4]{j^4}\cdot\sqrt[4]{j^4}\cdot\sqrt[4]{k^4}$

$\boxed{5j^2k}$

625
5 125
25 5
5 5

② $(625)^{1/4}\cdot(j^8)^{1/4}\cdot(k^4)^{1/4}$

$\sqrt[4]{625}\cdot j^2\cdot k$

$\boxed{5j^2k}$

d) $\dfrac{15d^2e^{2/3}f}{5df^{-4}}$

$\frac{15}{5}\cdot\frac{d^2}{d}\cdot\frac{e^{2/3}}{1}\cdot\frac{f\cdot f^4}{1}$

$3\cdot d\cdot e^{2/3}\cdot f^5$

$\boxed{3de^{2/3}f^5}$

e) $\dfrac{x^{2/3}}{x^{1/2}x^{3/4}}$

$\dfrac{x^{2/3}}{x^{\frac{2\cdot1}{2\cdot2}+\frac{3}{4}}} \rightarrow \dfrac{x^{2/3}}{x^{\frac{2}{4}+\frac{3}{4}}} \rightarrow \dfrac{x^{2/3}}{x^{5/4}}$

$x^{\frac{4\cdot2}{4\cdot3}-\frac{5\cdot3}{4\cdot3}}$

$x^{\frac{8}{12}-\frac{15}{12}}$

$x^{\frac{8-15}{12}}$

$x^{-7/12}$

$\boxed{\dfrac{1}{x^{7/12}}}$

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