Exponential Functions 7 – Solving Exponential Equations
Homework #12

Name ______________________
Per _____ **Date** ____________

1) Maria solved her problem wrong.

Method 2

Step 1:	$4^{x+1} = 8^x$
Step 2:	$\log_4 4^{x+1} = \log_4 8^x$
Step 3:	$x+1 = x\log_4 8$
Step 4:	$x+1 = 2x$
Step 5:	$1 = x$

a) Which step contains her mistake? Why?

Step 4

b) Solve the problem correctly.

1 $4^{x+1} = 8^x$

2 $\cancel{\log_4 4}^{x+1} = \log_4 8^x$

3 $x+1 = x \cdot \log_4 8$

$x+1 = x \cdot \frac{\log 8}{\log 4}$

4 $x+1 = 1.5x$

$-x \quad -x$

$\frac{1}{0.5} = \frac{0.5x}{0.5}$

5 $\boxed{x=2}$

In 2–10, solve the equation. Round to 3 decimal places. Check your answer.

2) $e^{-x} = 6$ 1

$\log e^{-x} = \log 6$

$\frac{-x \cdot \cancel{\log e}}{\cancel{\log e}} = \frac{\log 6}{\log e}$

$\frac{\cancel{-}x}{\cancel{-}1} = \frac{\frac{\log 6}{\log e}}{-1}$

[$\frac{\log(6)}{\log(e)}$ enter ÷ -1 enter]

$\boxed{x \approx -1.792}$

$e^{-(-1.792)} = 6$ ✓

3) $2^x = 15$ 2

$\cancel{\log_2 2}^x = \log_2 15$

$x = \frac{\log 15}{\log 2}$

$\boxed{x \approx 3.907}$

$2^{3.907} = 15$ ✓

4) $4^x - 5 = 3$ 1

$+5 \quad +5$

$4^x = 8$

$\log 4^x = \log 8$

$\frac{x \cdot \cancel{\log 4}}{\cancel{\log 4}} = \frac{\log 8}{\log 4}$

$\boxed{x = 1.5}$

5) $0.25^x - 0.5 = 2$

2

$+0.5 \quad +0.5$

$0.25^x = 2.5$

$\cancel{\log_{0.25}} \cancel{0.25^x} = \log_{0.25} 2.5$

$x = \frac{\log 2.5}{\log 0.25}$

$\boxed{x \approx -0.661}$

6) $1.2e^{-5x} + 2.\cancel{6} = 3$

1

$-2.\cancel{6} \quad -2.6$

$\frac{\cancel{1.2}e^{-5x}}{\cancel{1.2}} = \frac{0.4}{1.2}$

$e^{-5x} = \frac{\cancel{4}\ 1}{\cancel{12}\ 3}$

$e^{-5x} = \frac{1}{3}$

$\log e^{-5x} = \log \frac{1}{3}$

$\frac{-5x \cdot \cancel{\log e}}{\cancel{\log e}} = \frac{\log \frac{1}{3}}{\log e}$

$\frac{\cancel{-5}x}{\cancel{-5}} = \frac{\frac{\log \frac{1}{3}}{\log e}}{-5}$

$\left[\frac{\log\left(\frac{1}{3}\right)}{\log(e)} \text{ enter } \div -5 \text{ enter}\right]$

$\boxed{x \approx 0.220}$

7) $10^{3x-1} + \cancel{4} = 32$

2

$-\cancel{4} \quad -4$

$10^{3x-1} = 28$

$\cancel{\log_{10}} \cancel{10}^{3x-1} = \log_{10} 28$

$3x - \cancel{1} = \frac{\log 28}{\log 10} + 1$

$\cancel{+1}$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{\frac{\log 28}{\log 10} + 1}{3}$

$\left[\frac{\log(28)}{\log(10)} \text{ enter } +1 \text{ enter } \div 3 \text{ enter}\right]$

$\boxed{x \approx 0.816}$

8) $-5e^{-x}+9=6$ 2

$-9 \quad -9$

$\frac{\cancel{-5}e^{-x}}{\cancel{-5}}=\frac{-3}{-5}$

$e^{-x}=\frac{3}{5}$

$\cancel{\log_e} e^{-x}=\log_e\frac{3}{5}$

$\frac{\cancel{-x}}{-1}=\frac{\frac{\log\frac{3}{5}}{\log e}}{-1}$

$\left[\frac{\log(\frac{3}{5})}{\log(e)} \text{ enter} \div -1 \text{ enter}\right]$

$\boxed{x\approx 0.511}$

9) $3(4)^{2x}+1=5$ 1

$-1 \quad -1$

$\frac{\cancel{3}(4)^{2x}}{\cancel{3}}=\frac{4}{3}$

$4^{2x}=\frac{4}{3}$

$\log 4^{2x}=\log\frac{4}{3}$

$\frac{2x\cdot\cancel{\log 4}}{\cancel{\log 4}}=\frac{\log\frac{4}{3}}{\log 4}$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{\frac{\log\frac{4}{3}}{\log 4}}{2}$

$\left[\frac{\log(\frac{4}{3})}{\log(4)} \text{ enter} \div 2 \text{ enter}\right]$

$\boxed{x\approx 0.104}$

10) $36^{x-9}=6^{4x}$ 1

$\log 36^{x-9}=\log 6^{4x}$

$\frac{(x-9)\cdot\log 36}{\log 6}=\frac{(4x)\cdot\cancel{\log 6}}{\cancel{\log 6}}$

$(x-9)\cdot 2=4x$

$2x-18=4x$

$-2x \quad -2x$

$\frac{-18}{2}=\frac{\cancel{2}x}{\cancel{2}}$

$x=-9$

$y=a(1+r)^t$

$y=a(1-r)^t$

$A=P\left(1+\frac{r}{n}\right)^{nt}$

$*A=Pe^{rt}$

a = initial amount
r = rate (decimal)
t = time
P = principle

n = # of times compounded/year
annually = 1 quarterly = 4
semiannually = 2 monthly = 12

11) You deposit \$500 in an account that pays 2.5% annual interest compounded continuously. How long will it take for the balance to double?

$A=500(2)$
$=1000$

$A=Pe^{rt}$ find t

$P=500$

$r=2.5\%=0.025$

$$\frac{1000}{500}=\frac{\cancel{500}e^{0.025t}}{\cancel{500}}$$

$$2=e^{0.025t}$$

①

$\log 2=\log e^{0.025t}$

$$\frac{\log 2}{\log e}=\frac{0.025t\cdot\cancel{\log e}}{\cancel{\log e}}$$

$$\frac{\cancel{0.025}t}{\cancel{0.025}}=\frac{\frac{\log 2}{\log e}}{0.025}$$

$\left[\frac{\log(2)}{\log(e)}\text{ enter } \div 0.025 \text{ enter}\right]$

$t=27.726$

$\boxed{\text{About 28 years}}$

②

$\log_e 2=\cancel{\log_e e}^{0.025t}$

$$\frac{\cancel{0.025}t}{\cancel{0.025}}=\frac{\frac{\log 2}{\log e}}{0.025}$$

$t\approx 27.726$

$\boxed{\text{About 28 years}}$

12) The first permanent English colony in America was established in Jamestown, Virginia, in 1607. From 1620 through 1780, the population P (in thousands) of colonial America can be modeled by the equation $P = 8863(1.04)^t$ where t is the number of years since 1620. When was the population of colonial America about 345,000?

Find t

$P = 345,000$

$$\frac{345,000}{8863} = \frac{\cancel{8863}(1.04)^t}{\cancel{8863}}$$

$$\frac{345,000}{8,863} = 1.04^t$$

1

$$\log \frac{345,000}{8863} = \log 1.04^t$$

$$\frac{\log \frac{345,000}{8863}}{\log 1.04} = \frac{t \cdot \cancel{\log 1.04}}{\cancel{\log 1.04}}$$

$t \approx 93.360$

2

$$\log_{1.04}\left(\frac{345,000}{8863}\right) = \cancel{\log_{1.04}} \cancel{1.04}^t$$

$$t = \frac{\log\left(\frac{345,000}{8863}\right)}{\log 1.04}$$

$t \approx 93.360$

$1620 + 93$

In 1713, the population of colonial America was about 345,000.

13) You deposit $2000 in an account that pays 2% annual interest compounded quarterly. How long will it take for the balance to reach $2400?

$$2400=2000\left(1+\frac{0.02}{4}\right)^{4t}$$

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$

$n=4$

$P=2000$

$r=2\%=0.02$

$t=t$

$A=2400$

$$\frac{2400}{2000}=\frac{\cancel{2000}(1.005)^{4t}}{\cancel{2000}}$$

$$1.2=1.005^{4t}$$

1

$$\log 1.2=\log 1.005^{4t}$$

$$\frac{\log 1.2}{\log 1.005}=\frac{4t\cdot\log 1.005}{\log 1.005}$$

$$\frac{\cancel{4}t}{\cancel{4}}=\frac{\frac{\log 1.2}{\log 1.005}}{4}$$

$\left[\frac{\log(1.2)}{\log(1.005)}\right.$ enter $\div 4$ enter$\left.\right]$

$x\approx 9.139$

2

$$\log_{1.005}1.2=\cancel{\log_{1.005}1.005^{4t}}$$

$$\frac{\frac{\log 1.2}{\log 1.005}}{4}=\frac{\cancel{4}t}{\cancel{4}}$$

$x\approx 9.139$

It will take about 9.1 years / 9 years for the balance to reach $2400.