Module 12d: Applying Triangle Congruence Theorems

Math Practice(s):

-Reason abstractly & quantitatively.

-Construct viable arguments & critique the reasoning of others.

Learning Target(s):

-Use proofs to write convincing mathematical arguments.

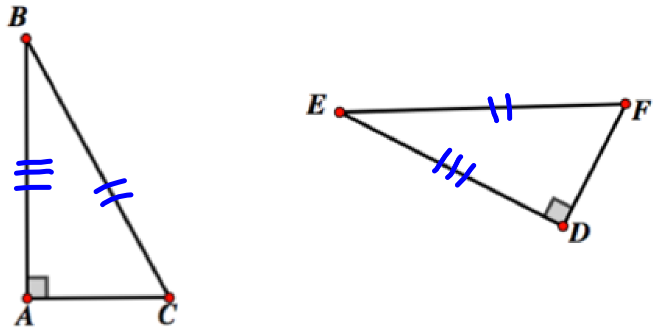
-Prove the perpendicular bisector thm & isosceles triangle thm.

Homework:

HW#13: 12d #1-4

The figures below are two right triangles with $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$.

$AB = DE \quad BC = EF$



1. Mark the diagram by placing "congruence marks" to show which parts are congruent.

2. What triangle congruence theorem could we try to use to prove these two triangles congruent?

SSA, but SSA is not a congruence thm...

One of the key facts of right triangles is the Pythagorean Theorem.

We can use this to solve for AC and DF.

$$\cancel{ED^2} + DF^2 = EF^2$$
$$-\cancel{ED^2} \qquad -ED^2$$
$$\cancel{\sqrt{}}DF^{\cancel{2}} = \sqrt{EF^2 - ED^2}$$
$$DF = \sqrt{EF^2 - ED^2}$$

$$\cancel{AB^2} + AC^2 = BC^2$$
$$-\cancel{AB^2} \qquad -AB^2$$
$$\cancel{\sqrt{}}AC^{\cancel{2}} = \sqrt{BC^2 - AB^2}$$
$$AC = \sqrt{BC^2 - AB^2}$$

Now, by substitution, [redacted] how can we conclude that $AC = DF$?

$$DF = \sqrt{EF^2 - ED^2} \qquad AB = DE \quad BC = EF$$
$$DF = \sqrt{BC^2 - AB^2}$$
$$DF = AC$$

we know

$\overline{DF} \cong \overline{AC}$

So, for right triangles, SSA can actually be turned into SSS, which is a triangle congruence theorem! What parts of the right triangle did we have to prove this?

we are given a hypotenuse & a leg.

The HYPOTENUSE – LEG (HL) Theorem

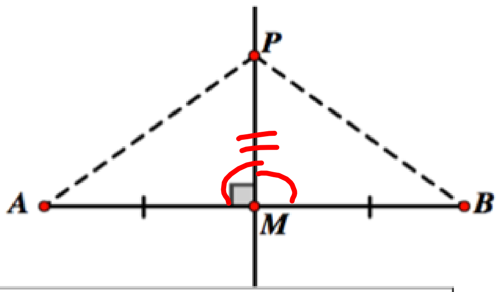
If the hypotenuse and leg of one **right** △ are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

Example 1:

The ***Perpendicular Bisector Theorem*** states the following:

Any point P on the perpendicular bisector of $\overline{AB}$ is equidistant (of equal distance) to its endpoints A and B.

Prove that the ***Perpendicular Bisector Theorem*** is true.



Given: Point P lies on the perpendicular bisector of $\overline{AB}$.

Prove: $AP = \cancel{PB}$ BP

	What statements can we make that must be true?	How do we know those statements must be true?
Part I	• P lies on ⊥ bis. of $\overline{AB}$	• Given
Part II	• $\overline{AM} \cong \overline{BM}$ • $\overline{PM} \cong \overline{PM}$ • $\angle AMP$ & $\angle BMP$ are rt $\angle$s. • $\angle AMP \cong \angle BMP$ • $\triangle PMA \cong \triangle PMB$ • $\overline{AP} \cong \overline{BP}$	• Def. of bisector • Reflexive Prop. • Def. of ⊥ • All rt $\angle$s $\cong$ • SAS • CPCTC
Part III	• $AP = BP$	• Def. of $\cong$

Example 2:

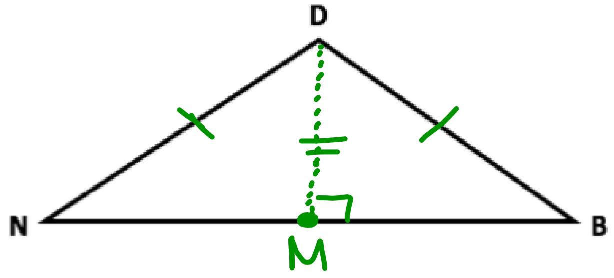
The ***Isosceles Triangle Theorem*** states the following:

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Prove that the ***Isosceles Triangle Theorem*** is true.

Given: In ΔDNB, $\overline{DN} \cong \overline{DB}$

Prove: $\angle N \cong \angle B$



	What statements can we make that must be true?	How do we know those statements must be true?
Part I	• $\overline{DN} \cong \overline{DB}$	• Given
Part II	• $\overline{DM}$ is $\perp$ bis. $\overline{NB}$ • $\overline{DM} \cong \overline{DM}$ • $\angle NMD$ & $\angle BMD$ are rt $\angle$s • $\triangle NMD$ & $\triangle BMD$ are rt $\triangle$s • $\triangle NMD \cong \triangle BMD$	• Converse of $\perp$ bisector thm • Reflexive Prop. • Def. of $\perp$ • Def. of rt $\triangle$ • HL
Part III	• $\angle N \cong \angle B$	• CPCTC

The Isosceles Triangle Theorem

If two sides of a triangle are $\cong$, then the angles opposite those sides are $\cong$.

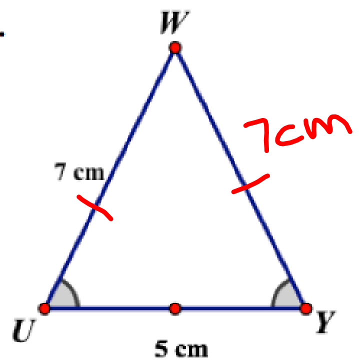
The CONVERSE of the Isosceles Triangle Theorem

If 2 $\angle$s of a $\triangle$ are $\cong$,

Then the sides opposite those $\angle$ s are $\cong$

Example 3: Determine the perimeter of each triangle.

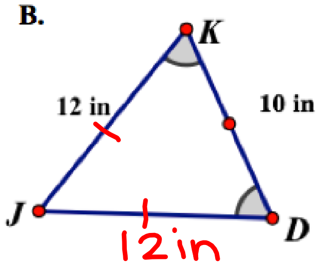
A.



$P=7+5+7$

$\boxed{P=19cm}$

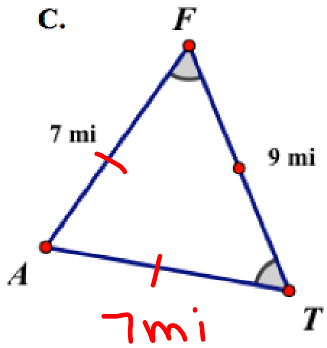
B.



$P=12+12+10$

$\boxed{P=34in}$

C.

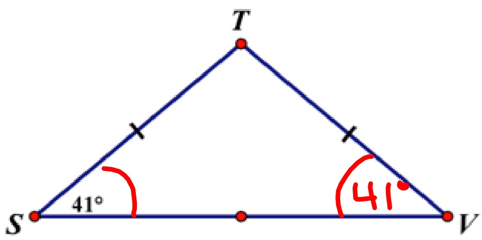


$P=7+7+9$

$\boxed{P=23mi}$

Example 4: Determine the measure of the two unknown angles in each triangle.

A.



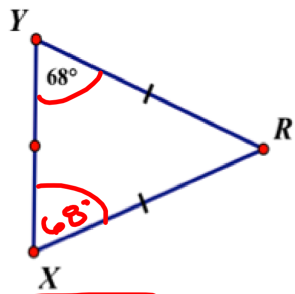
$\boxed{m\angle V=41^\circ}$

$41^\circ+41^\circ+m\angle T=180^\circ$

$82+m\angle T=180$
$-82 \qquad -82$

$\boxed{m\angle T=98^\circ}$

B.



$\boxed{m\angle X=68^\circ}$

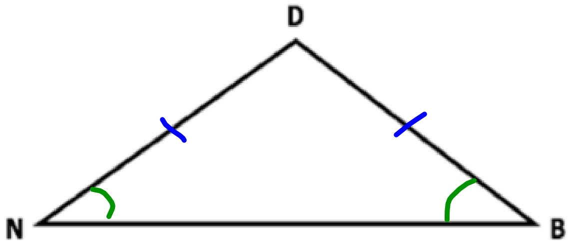
$68^\circ+68^\circ+m\angle R=180^\circ$

$136+m\angle R=180$
$-136 \qquad -136$

$\boxed{m\angle R=44^\circ}$

Practice

1. ΔDNB is an isosceles triangle with $\overline{DN} \cong \overline{DB}$.



A. If $DN = 5x - 31$, $NB = 45$ and $DB = 34$, determine the value of x.

$DN = DB$

$5x - 31 = 34$

$5x = 65$

$\boxed{x = 13}$

B. If $m\angle N = x + 20$ and $m\angle B = 90 - x$, determine the measure of each angle of ΔDNB.

$m\angle N = m\angle B$

$x + 20 = 90 - x$

$2x = 70$

$x = 35$

$m\angle N = m\angle B = 90 - (35)$

$\boxed{m\angle N = m\angle B = 55^\circ}$

$m\angle D + 55 + 55 = 180$

$\boxed{m\angle D = 70^\circ}$

C. If $DN = 3x$, $NB = 4x + 1$ and the perimeter of ΔDNB is 151, determine the length of each side of ΔDNB.

$DN = DB$

$P = DN + DB + NB$

$151 = 3x + 3x + 4x + 1$

$150 = 10x$

$x = 15$

$DN = DB = 3(15)$

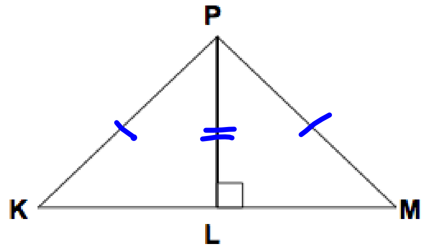
$\boxed{DN = DB = 45 \text{ units}}$

$NB = 4(15) + 1$

$\boxed{NB = 61 \text{ units}}$

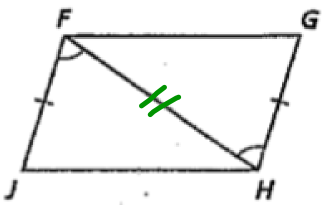
2. Given: ΔKPM is isosceles, $\overline{LP} \perp \overline{KM}$

Prove: $\Delta KLP \cong \Delta MLP$



	What statements can we make that must be true?	How do we know those statements must be true?
Part I	• ΔKPM is isosceles • $\overline{LP} \perp \overline{KM}$	• Given • Given
Part II	• $\overline{KP} \cong \overline{MP}$ • $\overline{PL} \cong \overline{PL}$ • $\angle KLP$ & $\angle MLP$ are rt $\angle s$ • ΔKLP & ΔMLP rt Δs	• Def. of Isosc. Δ • Reflexive Property • Def. of $\perp$ • Def. of rt. Δ
Part III	• $\Delta KLM \cong \Delta MLP$	• HL

3. Given: $\overline{FJ} \cong \overline{GH}$, $\angle JFH \cong \angle GHF$



Prove: $\overline{FG} \cong \overline{JH}$

	What statements can we make that must be true?	How do we know those statements must be true?
Part I	• $\overline{FJ} \cong \overline{GH}$ • $\angle JFH \cong \angle GHF$	• Given • Given
Part II	• $\overline{FH} \cong \overline{HF}$ • $\triangle JFH \cong \triangle GHF$	• Reflexive Prop. • SAS
Part III	• $\overline{FG} \cong \overline{JH}$	• CPCTC