Module 12c: Justifying Triangles are Congruent

Math Practice(s):
   -Reason abstractly & quantitatively.
   -Construct viable arguments & critique the reasoning of others.

Learning Target(s):
   -Use proofs to write convincing mathematical arguments.
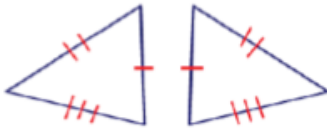
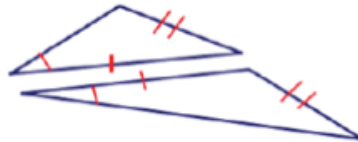**Homework:**
   HW#12: 12c #1-2

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Warm-up

Determine if each of the following pairs of triangles are congruent. If they are congruent, write a brief statement to explain how you know using one of the triangle congruence theorems to justify your conclusion.
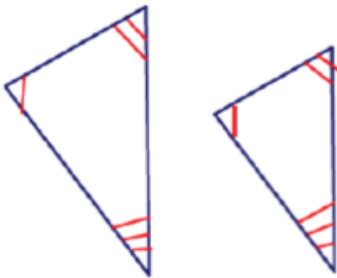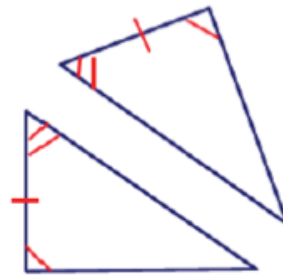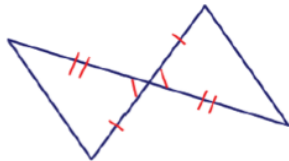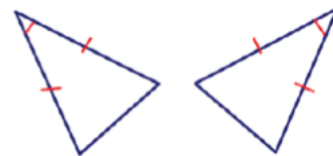
1.

SSS

2.

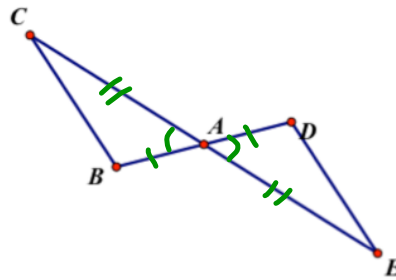Not ≅

3.

Not ≅ (but is ~)

4.

ASA

5.

SAS

6.

SAS

7.

AAS

Example 1:

In the figure to the right ($\triangle ABC$ and $\triangle ADE$),

- point A is the midpoint of $\overline{CE}$, and
- point A is the midpoint of $\overline{BD}$.

A. Based on the given information, what segments in the figure MUST be congruent? Explain why these segments must be congruent.

$$\overline{CA} \cong \overline{EA}$$
$$\overline{AB} \cong \overline{AD}$$

Definition of Midpoint

B. Based on your answers to question A, mark the diagram by placing "congruence marks" (tick marks) to show which segments are congruent.

C. Based on the given information, we can do some reasoning to figure out that there is one other pair of corresponding parts that must be congruent (either a pair of segments or a pair of angles). Determine which other pair of corresponding parts must be congruent (mark the diagram with the appropriate "congruence marks) and explain how you know.

$\angle CAB \cong \angle EAD$ vertical $\angle$s are congruent

D. Based on your answers above, which "triangle congruence theorem" (SSS, ASA, SAS, or SAA) could we use to prove that $\triangle ABC \cong \triangle ADE$?
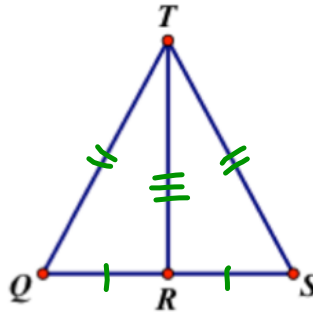
SAS

E. Your teacher heard another student in the class say that SSS could be used to prove the triangles are congruent. Explain why SSS is incorrect.

We don't know all the side lengths.

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Example 2:

In the figure to the right (ΔQRT and ΔSRT),

- point R is the midpoint of $\overline{QS}$, and
- $\overline{QT} \cong \overline{ST}$.

A. Based on the given information, which pair of segments in the figure MUST be congruent? Explain why these segments must be congruent.

$\overline{QR} \cong \overline{SR} \longrightarrow$ Def. of Midpoint

$\overline{QT} \cong \overline{ST} \longrightarrow$ Given

B. Based on your answers to question A, mark the diagram by placing "congruence marks" (tick marks) to show which segments are congruent.

C. Based on the given information, we can do some reasoning to figure out that there are other pairs of corresponding parts that must be congruent (either a pair of segments or a pair of angles). Determine which other pair of corresponding parts must be congruent (mark the diagram with the appropriate "congruence marks) and explain how you know.

$\overline{TR} \cong \overline{TR} \longrightarrow$ The Reflexive Property

D. Based on your answers above, which "triangle congruence theorem" (SSS, ASA, SAS, or SAA) could we use to prove that ΔQRT ≅ ΔSRT?

SSS

E. Your teacher heard another student in the class say that ASA could be used to prove the triangles are congruent because

- ∠QRT and ∠SRT are right angles so they are congruent, and
- ∠QTR and ∠STR must be congruent because $\overline{RT}$ cuts the big angle into 2 equal parts.

Do you agree or disagree with this student's conclusion? Explain why or why not?

Explanations vary.

Example 2: *(continued)*   **Try this page for HW over the weekend.**

The thought process we used as we went through questions A – D is an informal example of a proof.

You can think of a "proof" as having three parts:

Part I: Start with what you got
  • The GIVEN information are we starting with

Part II: Build the roadmap to the destination (keep the goal in mind)
  • What else can we figure out based on the given information ("one thing leads to another")

Part III: State where you arrived at (what you trying to prove -- the goal)

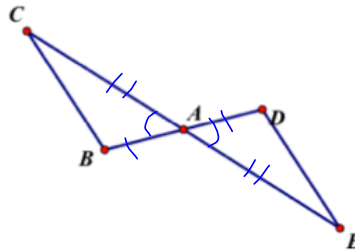Let's summarize our thought process as we went through questions A – D:

	What were we trying to do? What was our thought process?	What statements can we make that must be true?	How do we know those statements must be true?
Part I	State what must be obviously true based on the given information.	• Point R is midpoint $\overline{QS}$ • $\overline{QT} \cong \overline{ST}$	This information was **given** to us (we were told it was true).
Part II	Use what we previously learned (along with the given information) to figure out what else must be true.	• $\overline{RQ} \cong \overline{RS}$ • $\overline{TR} \cong \overline{TR}$	• Because of the **definition of a midpoint**. • The _Reflexive_ **Property** ($\overline{TR}$ is the same segment in both triangles, so it has to be congruent to itself)
Part III	Our destination (our goal) was to prove that the two triangles are congruent.	• $\triangle QRT \cong \triangle SRT$	Reason: _SSS_ *(based on what we stated must be true in Parts I and II, we need to determine why we can say for sure that we reached our destination)*

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Example 3:

Let's revisit example 1 and summarize Parts I – III of our thought process to prove $\triangle ABC \cong \triangle ADE$:

In the figure to the right ($\triangle ABC$ and $\triangle ADE$),

- point A is the midpoint of $\overline{CE}$, and
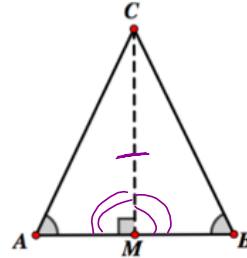- point A is the midpoint of $\overline{BD}$.

	What were we trying to do? What was our thought process?	What statements can we make that must be true?	How do we know those statements must be true?
Part I	State what must be obviously true based on the given information.	• A is midpoint $\overline{CE}$ • A is midpoint $\overline{BD}$	This information was **given** to us (we were told it was true).
Part II	Use what we previously learned (along with the given information) to figure out what else must be true.	• $\overline{CA} \cong \overline{EA}$ • $\overline{AB} \cong \overline{AD}$ • $\angle CAB \cong \angle EAD$	• Def. of midpoint • Def. of midpoint • Vertical $\angle$s $\cong$
Part III	Our destination (our goal) was to prove that the two triangles are congruent.	• $\triangle ABC \cong \triangle ADE$	Reason: ___SAS___ *(based on what we stated must be true in Parts I and II, we need to determine why we can say for sure that we reached our destination)*

Example 4:

Given: In $\triangle ABC$, $\angle A \cong \angle B$ and $\overline{CM} \perp \overline{AB}$.

Prove: $\overline{AC} \cong \overline{BC}$

	What were we trying to do? What was our thought process?	What statements can we make that must be true?	How do we know those statements must be true?
Part I	State what must be obviously true based on the given information.	• $\angle A \cong \angle B$ • $\overline{CM} \perp \overline{AB}$	This info. was **given** to us (we were told it was true).
Part II	Use what we previously learned (along with the given information) to figure out what else must be true.	• $\angle AMC$ & $\angle BMC$ are right $\angle s$ • $\angle AMC \cong \angle BMC$ • $\overline{CM} \cong \overline{CM}$ • $\triangle AMC \cong \triangle BMC$	• Def. of $\perp$ • All rt $\angle s \cong$ • Reflexive Prop. • AAS
Part III	Our destination (our goal) was to prove that the two **SEGMENTS** are congruent.	• $\overline{AC} \cong \overline{BC}$	Reason: CPCTC *(based on what we stated must be true in Parts I and II, we need to determine why we can say for sure that we reached our destination)*

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