Module 12b: Triangle Congruence Theorems

Math Practice(s):

-Reason abstractly & quantitatively.

-Construct viable arguments & critique the reasoning of others.

Learning Target(s):

-Understand that Corresponding Parts of Congruent Triangles are Congruent (CPCTC).

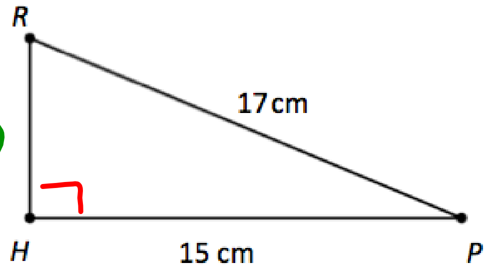
-Determine which corresponding pairs of angles & sides are necessary to guarantee congruent triangles.

Homework:

HW#11: 12b #1-7

Warm-up

1. In the figure to the right, $\overline{RH} \perp \overline{HP}$.



A. Determine the area of ΔRHP.

① $15^2 + b^2 = 17^2$

$RH = 8\text{ cm}$

② $\frac{1}{2}(8)(15)$

$\boxed{A = 60\text{ cm}^2}$

B. Determine the measure of $\angle R$.

$\sin^{-1}\left(\frac{15}{17}\right) = m\angle R$

$\boxed{m\angle R = 62^\circ}$

C. If a rigid motion transformation was performed on ΔRHP to create ΔKLM where $\Delta RHP \cong \Delta KLM$, determine the following measurements:

$LM =$ 15 cm $\quad m\angle L =$ 90° $\quad m\angle M =$ 28°

D. ΔRHP was dilated about point H by a scale factor 5 to create $\Delta R'HP'$. Determine the **perimeter** and the **area** of $\Delta R'HP'$.

$A_{R'HP'} = 5^2(60)$

$= 25(60)$

$\boxed{A = 1500\text{ cm}^2}$

$P_{RHP} = 8 + 15 + 17 = 40\text{ cm}$

$P_{\Delta R'HP'} = 40(5)$

$\boxed{P = 200\text{ cm}}$

Corresponding Parts of Congruent Triangles are Congruent (CPCTC) (#THM):

If two triangles are congruent, then their six corresponding parts (their 3 angle pairs and their 3 side pairs) must also be congruent.

The converse of "CPCTC"

If ΔABC and ΔPQR have

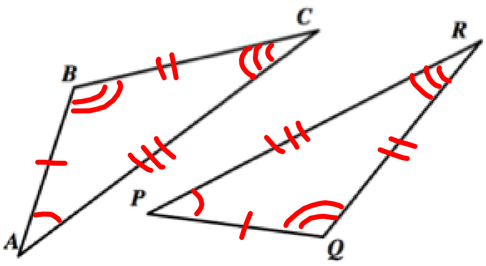
all pairs of corresponding sides congruent:

$\overline{AB} \cong \overline{PQ}$
$\overline{BC} \cong \overline{QR}$
$\overline{AC} \cong \overline{PR}$

all pairs of corresponding angles congruent:

$\angle A \cong \angle P$
$\angle B \cong \angle Q$
$\angle C \cong \angle R$

then, $\Delta ABC \cong \Delta PQR$.



We've just found that 6 pieces of information is enough to conclude that two triangles are congruent, but can we use fewer pieces to prove triangles congruent?

Consider ΔABC, with angle measures 12°, 133°, and 35° respectively. If ΔRST has angles with the same measures, can we conclude ΔABC ≅ ΔRST? Explain why or why not.

No, because you don't know the measures of the side lengths, so it could be bigger or smaller with the same ∠ measures.

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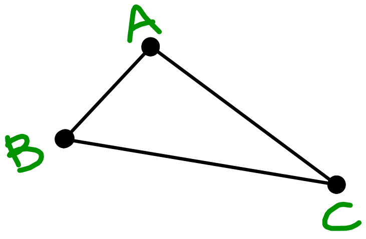
WHAT DOES IT TAKE TO BE CONGRUENT?

Scenario 1: Side-Side-Side (SSS)

In this scenario, you will explore if having three sides of one triangle congruent to three sides of another triangle guarantees that the two triangles are congruent.

1. Draw a scalene triangle in the space to the right, and label it ΔABC. Measure the sides of the triangle, and label the triangle.



2. Now, have your group member draw 3 segments on separate pieces of patty paper with the same lengths as the sides of your triangle. Your partner should put the 3 segments together to form a new triangle. Tape your patty paper triangle together.

3. Slide your partners patty paper triangle over your original triangle above, reflecting and rotating as needed, to line up their patty paper triangle with your original triangle. What do you notice?

They line up almost perfectly.

4. Check to see if the other members in your group made the same conjecture. Explain why this happened.

The sides can only match up in 1 way

5. Could another group member rearrange the patty paper segments to create a new triangle that is not congruent to the original? Explain why the two triangles must be congruent, or why not.

No, no matter how we rearranged the sides, we could always do a rigid motion transformation to line them up.

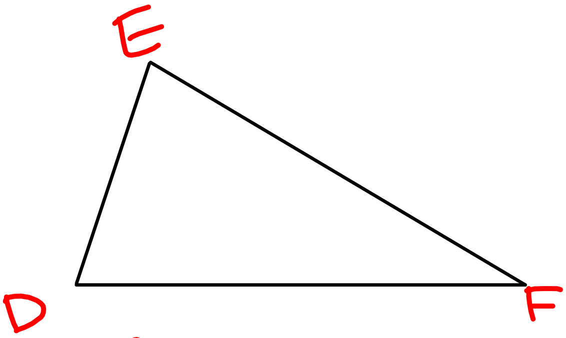
Conjecture:

If three sides of one triangle are congruent to three sides of another triangle, then

the triangles are congruent.(#THM)

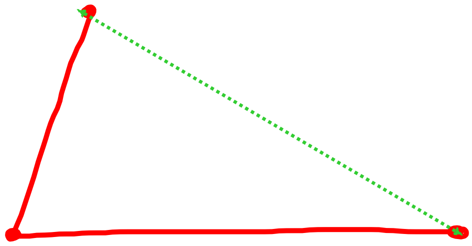
Scenario 2: Side-Angle-Side (SAS)

In this scenario, you will explore if having two sides and the angle between them of one triangle congruent to two sides and the angle between them of another triangle guarantees that the two triangles are congruent.



1. Draw a scalene triangle in the space to the right, and label it ΔDEF. Measure the sides of the triangle, and label the triangle.

2. Have a group member trace 2 segments and the created angle of your triangle onto a sheet of patty paper. This group member should now form a triangle from the segments and traced angle, and a third side drawn in to close the triangle.



3. Slide your partners patty paper triangle over your original triangle above, reflecting and rotating as needed, to line up their patty paper triangle with your original triangle. What do you notice?

They match up.

4. Is the new triangle congruent to the original? Explain why or why not.

Yes, the sides & angles are congruent.

5. Can you rearrange the pieces to create a new triangle that is not congruent to the original, where the angle is still between the two sides? Explain why the two triangles must be congruent, or why not.

No, they always line up with the original △.

Conjecture:

If two sides and the angle between the sides of one triangle are congruent to two sides and the angle between the sides of another triangle, then

the triangles are congruent. (#THM)

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Scenario 3: Side-Side-Angle (SSA)

In this scenario you will explore if having two adjacent sides and the angle adjacent to the second side of one triangle congruent to two adjacent sides and the angle adjacent to the second side of another triangle guarantees that the two triangles are congruent.

Conjecture:

If two adjacent sides and the angle adjacent to the second side of one triangle are congruent to two adjacent sides and the angle adjacent to the second side of another triangle, then

the triangles are not congruent.

Scenario 4: Side-Angle-Angle (SAA) aka (AAS)

In this scenario you will explore if having two angles and the side not between them of one triangle congruent to two angles and one of the sides not between them, of another triangle guarantees that the two triangles are congruent.

Conjecture:

If two angles and the side not between them of one triangle are congruent to two angles and the side not between them of another triangle, then

the triangles are congruent.(#THM)

Scenario 5: Angle-Side-Angle (ASA)

In this scenario you will explore if having two angles and the side between them of one triangle congruent to two angles and the side between them of another triangle guarantees that the two triangles are congruent.

Conjecture:

If two angles and the side between them of one triangle are congruent to two angles and the side between them of another triangle, then

the triangles are congruent.(#THM)

Scenario 6: Angle-Angle-Angle (AAA)

On patty paper, draw a triangle with angle measures of _______, _______, and _______. Compare your triangle with at least 3 people around you. What do you notice about triangles that have 3 pairs of congruent corresponding angles?

Conjecture:

If three angles of one triangle are congruent to three angles of another triangle, then

the triangles are not congruent.

Triangle Congruence Summary

Complete the following. Use this summary sheet as a resource in class and for homework.

- List the four Congruence Theorems here. Write the acronym and then describe what that acronym means. Be specific and clear when describing an angle or side.

- Angle-Side-Angle (ASA)
  2 ∠s & side between the 2 ∠s.
- Side-Side-Side (SSS)
  3 pairs of sides
- Angle-Angle-Side (AAS)
  2 ∠s & side not between 2 ∠s.
- Side-Angle-Side (SAS)
  2 sides & the ∠ between the sides.

- By definition, congruent triangles have ≅ corresponding sides and ≅ corresponding ∠s.

- Thus, we can say "Corresponding parts of congruent triangles are congruent." (#THM) We use this statement very often in geometry. When we use it, we use an acronym, CPCTC.

Congruence Statement

If ΔABC is congruent to ΔDEF, then we write $\Delta ABC \cong \Delta DEF$.

So, if $\Delta ABC \cong \Delta DEF$ then complete the following:

$\angle E \cong$ ∠B	$\overline{AB} \cong \overline{DE}$
$\angle C \cong$ ∠F	$\overline{EF} \cong \overline{BC}$
$\angle D \cong$ ∠A	$\overline{AC} \cong \overline{DF}$

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