Polynomial 6b - Graphing Polynomials in Standard Form

Standards: A-APR.2, A-APR.6, A-REI.4

GLO: #3 Complex Thinker

Math Practice: #1 - Make sense of problems and persever in solving them

Learning Target:
How can we use division to graph polynomials?

Oct 30-9:18 PM

Warm Up:

Find the zeros by factoring:

a) $f(x) = 10x^2 - 26x - 12$

GCF = 2

$5x^2 - 13x - 6$ a = 5, b = -13, c = -6

a·c = -30 b = -13

1 -30
2 -15
3 -10
5 -6

	5x	+2
x	$5x^2$	+2x
-3	-15x	-6

$2(5x+2)(x-3) = 0$

$\cancel{2 \neq 0}$ $5x + 2 = 0$ $x - 3 = 0$

$-2 \quad -2$ $+3 \quad +3$

$5x = -2$ $x = 3$

$x = -\frac{2}{5}$

$\boxed{\text{zeros: } 3, -\frac{2}{5}}$

b) $f(x) = 2x^2 + 2x - 40$

GCF = 2

$x^2 + x - 20$ a = 1, b = 1, c = -20

a·c = -20 b = 1

-1 20
-2 10
-4 5

	x	-4
x	x^2	-4x
+5	+5x	-20

$2(x-4)(x+5) = 0$

$2 = 0$ $x - 4 = 0$ $x + 5 = 0$

$\boxed{\text{zeros: } -5, 4}$

Find the zeros by using the Quadratic Formula:

c) $f(x) = 9x^2 - 8x - 10$

a = 9
b = -8
c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(9)(-10)}}{2(9)}$$

$$x = \frac{8 \pm \sqrt{64 + 360}}{18}$$

$$x = \frac{8 \pm \sqrt{424}}{18}$$

$\frac{8 + \sqrt{424}}{18}$ $\frac{8 - \sqrt{424}}{18}$

1.588 -0.6995

-0.700

$\boxed{\text{zeros: } 1.588 \text{ \& } -0.700}$

x-int: (1.588, 0) & (-0.700, 0)

Summary of the last few lessons (erase to show)

1. **Standard** form of a polynomial function is

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$$

$$P(x) = -2x^4 + 1x^3 - 4x^2 - 17x + 5$$

2. The degree & leading coefficient yields **end behavior** and the constant term yields the **y-intercept** .

3. The **zeros** (including multiplicity) often reveal if a polynomial changes direction.

Now that we know how to factor polynomials given at least one zero or factor and using long division, we proceed by finding the remaining zeros, which will allow us to sketch a rough graph that includes all intercepts. Note: it is possible that some zeros will be complex, which will not show up on the graph, but which still tells us the number of x-intercepts.

Ex 1 x = 1 is a zero of the polynomial

factor is (x−1) ✓

$$f(x) = x^3 - 6x^2 + 11x - 6$$

Use polynomial long division to rewrite it in factored form.

remainder is 0.

$$
\begin{array}{r|l}
 & x^2 - 5x + 6 \\
\hline
x-1 & x^3 - 6x^2 + 11x - 6 \\
 & +(-x^3 + x^2) \\
\hline
 & -5x^2 + 11x - 6 \\
 & +(+5x^2 - 5x) \\
\hline
 & 6x - 6 \\
 & +(-6x + 6) \\
\hline
 & 0
\end{array}
$$

$f(x) = (x-1)(x^2 - 5x + 6)$ a = 1, b = −5, c = 6

a·c = 6 b = −5

1 6

−2 −3

	x	−2
x	x^2	−2x
−3	−3x	+6

$$\boxed{f(x) = (x-1)(x-2)(x-3)}$$

Now we can use the zero product property & what we know about solving quadratics to find the remaining zeros to graph. Your graph of f(x) should accurately show the location of all x- and y-intercepts; sketch the general shape of the graph through these points.

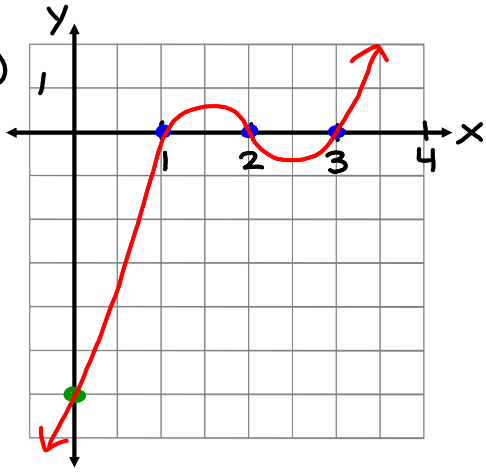
$$f(x) = x^3 - 6x^2 + 11x - 6$$

$f(x) = (x-1)(x-2)(x-3)$

x-int: (1,0) (2,0) (3,0)

y-int: (0,−6)

EB: ↙ ↗



2: Given that $x = -1$ is a zero of $P(x) = x^3 - 5x^2 + 6$ determine all x- and y-intercepts of P(x). Then, sketch the general shape of the graph of P(x) through these points.

(x+1)

① long division

$$\begin{array}{r} x^2 - 6x + 6 \\ x+1 \overline{) x^3 - 5x^2 + 0x + 6} \\ +(-x^3 - x^2) \\ \hline -6x^2 + 0x + 6 \\ +(+6x^2 + 6x) \\ \hline 6x + 6 \\ +(-6x - 6) \\ \hline 0 \end{array}$$

$P(x) = (x+1)(x^2 - 6x + 6)$ a = 1, b = −6, c = 6

② Factor, if possible

$a \cdot c = 6$ $b = -6$

−1 · −6, −2 · −3

nothings adds up to −6, so you can't factor.

③ Factored Form

$$\boxed{P(x) = (x+1)(x^2 - 6x + 6)}$$

x-int: (−1, 0)
(4.732, 0)
(1.268, 0)

y-int: (0, 6)

EB: ↙ ↗

$x^2 - 6x + 6 = 0$ a = 1, b = −6, c = 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

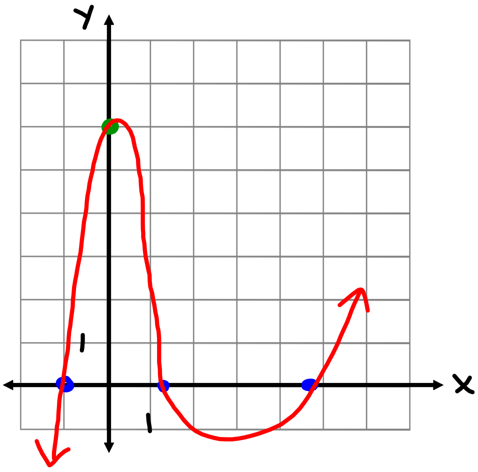
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{2}$$

$$x = \frac{6 \pm \sqrt{12}}{2}$$

$\frac{6 + \sqrt{12}}{2}$ $\frac{6 - \sqrt{12}}{2}$

4.732 1.268



3: Given that $(2x-1)$ is a factor of

$$K(x)=2x^3-x^2-8x+4$$

determine all x- and y-intercepts of K(x). Then, sketch the general shape of the graph of K(x) through these points.

① long division

$$\begin{array}{r} x^2 - 4 \\ 2x-1 \overline{)\, \cancel{2x^3} - \cancel{x^2} - 8x + 4} \\ +(\cancel{-2x^3} + \cancel{x^2}) \downarrow \quad \downarrow \\ \hline -\cancel{8x} + \cancel{4} \\ +(+\cancel{8x} - \cancel{4}) \\ \hline 0 \end{array}$$

$K(x)=(2x-1)\underbrace{(x^2-4)}$

② Factor, if possible

x^2-4 ← difference of squares

$(x+2)(x-2)$ $a^2-b^2=(a+b)(a-b)$

③ Factored Form

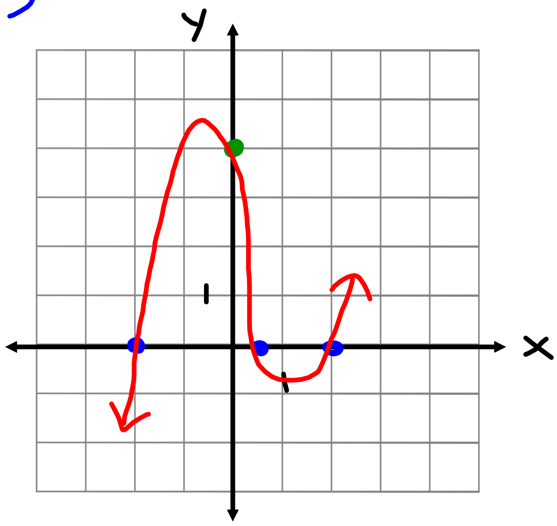
$$\boxed{K(x)=(2x-1)(x+2)(x-2)}$$

x-int: $(\frac{1}{2},0)(-2,0)(2,0)$

$2x-1=0$ $x+2=0$ $x-2=0$

y-int: $(0,4)$

EB: ↙ ↗



4: Given that $(-5,0)$ is an x-intercept of zero @ −5

$$g(x) = x^3 + 6x^2 + 3x - 10$$ factor $(x+5)$

determine all intercepts of g(x). Then, sketch the general shape of the graph of g(x) through these points.

① long division

$$\begin{array}{r} x^2 + x - 2 \\ x+5 \overline{) x^3 + 6x^2 + 3x - 10} \\ +(-x^3 - 5x^2) \downarrow \quad \downarrow \\ \hline x^2 + 3x - 10 \\ +(-x^2 - 5x) \downarrow \\ \hline -2x - 10 \\ +(+2x + 10) \\ \hline 0 \end{array}$$

$g(x) = (x+5)(x^2 + x - 2)$

a = 1
b = 1
c = −2

② Factor, if possible.

a·c = −2 b = 1

−1 2

	x	−1
x	x^2	−1x
+2	+2x	−2

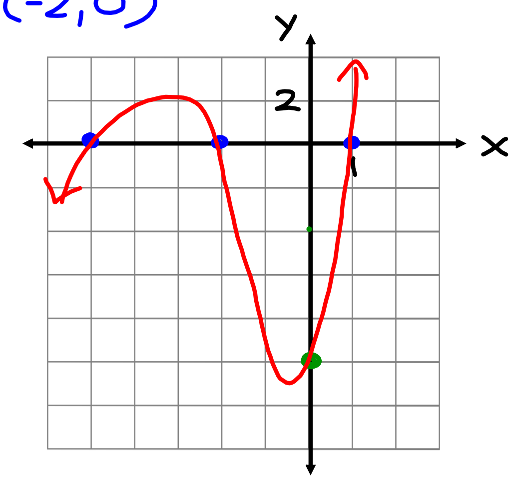
③ Factored Form

$$\boxed{g(x) = (x+5)(x-1)(x+2)}$$

x-int: (−5,0), (1,0), (−2,0)

y-int: (0,−10)

EB: ↙ ↗



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