Polynomial 6a - Polynomial Long Division

Standards: A-APR.2, A-APR.6, A-REI.4

GLO: #3 Complex Thinker

Math Practice: #7 Look for and make use of structure

Learning Target:

How is long division used with polynomials?

Warm up: Use long division to divide

$$\frac{875}{5} = 175$$

$$\begin{array}{r} 175 \\ 5\,\overline{)\,875} \\ -5 \\ \hline 37 \\ -35 \\ \hline 25 \\ -25 \\ \hline 0 \end{array}$$

$$\frac{93}{7} = 13\frac{2}{7}$$

$$\begin{array}{r} 13 \text{ r}2 \\ 7\,\overline{)\,93} \\ -7 \\ \hline 23 \\ -21 \\ \hline 2 \end{array}$$

Vocabulary Review:

(erase to show)

- The expression in the denominator or outside the long division symbol is called the **divisor** .

- The expression in the numerator or inside the long division symbol is called the **dividend** .

- The answer is called the **quotient** .

(erase to show)

The Factor (or Remainder) Theorem:

$x = r$ is a zero of the polynomial function f if and only if **$(x - r)$** is a factor of f.

So, the polynomial function f has an x-intercept at $(r, 0)$ if and only if $(x - r)$ is a factor of f.

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Knowing the zeros for a polynomial makes it easy to roughly sketch its graph, and the graph in turn allows us to better interpret the meaning of the symbolic form.

Finding zeros is *critical* for understanding the behavior of a polynomial, particularly its changes in direction.

This was true for quadratic functions as well and in previous lessons we were able to easily locate zeros because we could visually locate them on a given graph or because the symbolic forms were given in factored form.

Now we need a method to find the zeros of a polynomial function given in standard form IF we know at least one zero for the function.

This method is called **Polynomial Long Division**

(erase to show)

(erase to show)

Example 1: Rewrite $f(x) = x^3 - 4x^2 + 2x + 3$ in factored form if we know $x = 3$ is one of the zeros.

Notice: $x = 3$ is a zero means **$(x - 3)$** is a factor.

So how do we determine the other factor(s)?

$$(x-3)(\_\_\_\_?\_\_\_\_\_) = x^3 - 4x^2 + 2x + 3$$

We can use long division!

$$\frac{x^3 - 4x^2 + 2x + 3}{x - 3} \qquad \textit{Original Problem}$$

$$x - 3\overline{\big)\, x^3 - 4x^2 + 2x + 3} \qquad \textit{Rewrite in long division form}$$

For this problem we first want to determine what we need to multiply *(x – 3)* by to get x^3. Of course the answer is x^2, but when we multiply x – 3 by x^2 we get more than x^3. Subsequent steps are given below.

$$\begin{array}{rl}
x^2 & \textit{Place } x^2 \textit{ in the quotient} \\
x - 3\overline{\big)\, x^3 - 4x^2 + 2x + 3} & \\
-(x^3 - 3x^2) & \textit{Multiply } x^2 \textit{ by } x-3 \textit{ and subtract} \\
\hline
-x^2 + 2x + 3 &
\end{array}$$

$$\begin{array}{rl}
x^2 - x & \\
x - 3\overline{\big)\, x^3 - 4x^2 + 2x + 3} & \textit{We now need } -x^2\textit{, so we place } -x \textit{ in the quotient} \\
-(x^3 - 3x^2) & \\
\hline
-x^2 + 2x + 3 & \\
-(-x^2 + 3x) & \textit{Multiply } -x \textit{ by } x-3 \textit{ and subtract} \\
\hline
-x + 3 &
\end{array}$$

Continuing, we finally place -1 in the quotient to get

$$\begin{array}{rl}
x^2 - x - 1 & \\
x - 3\overline{\big)\, x^3 - 4x^2 + 2x + 3} & \\
-(x^3 - 3x^2) & \\
\hline
-x^2 + 2x + 3 & \\
-(-x^2 + 3x) & \textit{We now need } -x\textit{, so we place } -1 \textit{ in the quotient} \\
\hline
-x + 3 & \\
-(-x + 3) & \textit{Multiply } -1 \textit{ by } x-3 \textit{ and subtract} \\
\hline
0 &
\end{array}$$

Since the remainder is zero we know that $(x - 3)$ evenly divides $x^3 - 4x^2 + 2x + 3$, which implies that $(x^2 - x - 1)$ is a *factor* of $x^3 - 4x^2 + 2x + 3$.

In other words,

$$x^3 - 4x^2 + 2x + 3 = (x-3)(x^2 - x - 1)$$

Later, when we need to graph, we can use the methods we already know (Factoring or Quadratic Formula) to find the other zeros of $x^2 - x - 1$.

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Caution: Notice the use of parentheses throughout the subtraction steps. When performing long division with numbers there is usually no need for parentheses, but since polynomials have multiple terms that need to be subtracted from another polynomial, parentheses are essential. Below is an example of what to do when the dividend does not contain all powers of x.

Example 2: Use Polynomial Long Division to rewrite

$f(x) = 2x^3 - 7x + 5$ in factored form if x = 1 is a zero.

remainder should be 0.

$(x-1)$

$$
\begin{array}{r}
2x^2+2x-5 \\
x-1\overline{)\,2x^3+0x^2-7x+5} \\
+(-2x^3+2x^2) \downarrow \quad \downarrow \\
\hline
2x^2-7x+5 \\
+(-2x^2+2x) \downarrow \\
\hline
-5x+5 \\
+(+5x-5) \\
\hline
0
\end{array}
$$

$2x^3-7x+5 = (x-1)(2x^2+2x-5)$

$\boxed{f(x) = (x-1)(2x^2+2x-5)}$

Practice:

Use Polynomial Long Division to find the quotient of

$$\frac{x^3+3x^2-13x+6}{x-2} = \boxed{x^2+5x-3}$$

$$\begin{array}{r}
x^2+5x-3 \\
x-2\,\big)\,\overline{\cancel{x^3}+3x^2-13x+6} \\
+(\cancel{-x^3}+2x^2) \downarrow \quad \downarrow \\
\hline
\cancel{5x^2}-13x+6 \\
+(\cancel{-5x^2}+10x) \quad \downarrow \\
\hline
\cancel{-3x}+\cancel{6} \\
+(+\cancel{3x}-\cancel{6}) \\
\hline
0
\end{array}$$

Since $(x-2)$ evenly divides $x^3+3x^2-13x+6$ (no remainder), it follows that $(x-2)$ is a factor of $x^3+3x^2-13x+6$.

Using your work above, rewrite $x^3+3x^2-13x+6$ in factored form:

$$\boxed{(x-2)(x^2+5x-3)}$$

Example 3: Use Polynomial Long Division to rewrite (in factored form)

$f(x) = 4x^3 + 2x^2 - 4x + 3$ if $2x+3$ is a factor.

remainder = 0

Remember: "factor" means the remainder should be zero!

$$\begin{array}{r l}
 & 2x^2 - 2x + 1 \\
2x+3 \overline{)} & 4x^3 + 2x^2 - 4x + 3 \\
 & +(-\cancel{4x^3} - 6x^2) \\
\hline
 & -\cancel{4x^2} - 4x + 3 \\
 & +(+\cancel{4x^2} + 6x) \\
\hline
 & \cancel{2x} + \cancel{3} \\
 & +(-\cancel{2x} - \cancel{3}) \\
\hline
 & 0
\end{array}$$

$f(x) = 4x^3 + 2x^2 - 4x + 3$

$$\boxed{f(x) = (2x+3)(2x^2 - 2x + 1)}$$

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Example 4: Use Polynomial Long Division to find the quotient:

$$\frac{x^3 - 1}{x^2 + x + 1} = x - 1$$

$$x - 1$$

$$x^2 + x + 1 \overline{)\, \cancel{x^3} + 0x^2 + 0x - 1}$$

$$+\left(\cancel{-x^3} - 1x^2 - 1x\right)$$

$$-\cancel{1x^2} - \cancel{1x} - \cancel{1}$$

$$+\left(+\cancel{1x^2} + \cancel{1x} + \cancel{1}\right)$$

$$0$$

Example 5: Use Polynomial Long Division:

$$\left(5x^3+12x^2+7x-1\right)\div\left(x+2\right)$$

$$
\begin{array}{r|l}
 & 5x^2+2x+3 \\
\hline
x+2 & 5\cancel{x^3}+12x^2+7x-1 \\
 & +\left(-5\cancel{x^3}-10x^2\right) \downarrow \quad \downarrow \\
\hline
 & 2\cancel{x^2}+7x-1 \\
 & +\left(-2\cancel{x^2}-4x\right) \downarrow \\
\hline
 & 3\cancel{x}-1 \\
 & +\left(-3\cancel{x}-6\right) \\
\hline
 & -7
\end{array}
$$

$$\boxed{5x^2+2x+3+\frac{-7}{x+2}}$$

What if your remainder was not zero?

Short answer: then the divisor is NOT a factor of the dividend, and setting the divisor to zero will not give you a zero for the dividend.

Remember, only when the remainder is zero do we know *(x – r)* to be a factor.

(last slide)

Practice: Divide:

$$\left(x^3+3x^2-12x-1\right)\div\left(x+5\right)$$

$$
\begin{array}{r l}
 & \quad x^2-2x-2 \\
x+5 \,\big) & \overline{\,\cancel{x^3}+3x^2-12x-1} \\
+ & \underline{(-\cancel{x^3}-5x^2)} \\
 & \quad -\cancel{2x^2}-12x-1 \\
+ & \quad \underline{(+\cancel{2x^2}+10x)} \\
 & \qquad -\cancel{2x}-1 \\
+ & \qquad \underline{(+\cancel{2x}+10)} \\
 & \qquad\qquad 9
\end{array}
$$

$$\boxed{x^2-2x-2+\frac{9}{x+5}}$$