

Radicals 2 - Solving Radical Equations

Standard: A-REI.2

Math Practice: Look for and make use of structure

GLOs: #3-Complex Thinker

HW: WS#8

Learning Target:

- How do you solve a radical equation?
- How do you know if the solution is extraneous?

Warm Up: Solve for x :

a) $x^2 - 7x = -12$
 $+12 +12$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$x-3=0 \quad x-4=0$$

$+3 +3 \quad +4 +4$

$$x=3 \quad x=4$$

b) $5x^2 + 8 = 53$
 $-8 -8$

$$\frac{5x^2}{5} = \frac{45}{5}$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

In this unit, we will examine how to solve **square root equations**. These are equations that have x inside a square root

Steps to Solve a Radical Equation:

1. Get the square root expression by itself on one side of the equation
2. **Square** both sides (to cancel out the radical)
3. Solve the remaining equation using previously learned techniques (typically those involving linear or quadratic equations).
4. Check your solution(s)!*

Whenever you square both sides of an equation, you technically get a new (but related) equation. And sometimes, when you do this, you obtain a solution that may work for the “new” equation, but **does not work** for the original equation. Such answers are

called **extraneous** **solutions** .

So ALWAYS check your answers!

Example 1: Solve for x :

$$\sqrt{x-1} = 3$$

$$x-1 = 9$$

$$x = 10$$

Square both sides of the equation

Add 1 to both sides of the equation

Check: $\sqrt{10-1} = 3$

$$\sqrt{9} = 3$$

$$3 = 3 \quad \checkmark$$

So, our solution is $x = 10$.

Practice: Solve for x :

$$\text{a) } \sqrt{3x+4} = (4)^2$$

$$\text{③ } 3x+4 = 16$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

$$\text{④ } \sqrt{3(4)+4} = 4$$

$$\sqrt{12+4} = 4$$

$$\sqrt{16} = 4$$

$$4 = 4 \quad \checkmark$$

$$\boxed{x=4}$$

$$\text{b) } \sqrt{2x-1} = (-2)^2$$

$$\text{③ } 2x-1 = 4$$

$$\frac{2x}{2} = \frac{5}{2}$$

$$x = \frac{5}{2}$$

extraneous solution

$$\text{④ } \sqrt{2\left(\frac{5}{2}\right)-1} = -2$$

$$\sqrt{5-1} = -2$$

$$\sqrt{4} = -2$$

$$2 = -2 \quad \times$$

No Solution

Practice: Solve for x:

①
②
c) $(x)^2 = \sqrt{x+6}$

③ $x^2 = x+6$
 $-x \quad -x \quad -6$
 -6
 $x^2 - x - 6 = 0$
 $a=1 \quad b=-1 \quad c=-6$
 $a \cdot c = -6 \quad b = -1$
 $1 \quad 6$
 $2 \quad -3$

	x + 2	
x	x ²	+2x
-3	-3x	-6

$(x+2)(x-3) = 0$
 $x+2=0 \quad x-3=0$
 $-2 \quad -2 \quad +3 \quad +3$
 ~~$x = -2$~~ $x = 3$

④ $(-2) = \sqrt{(-2)+6}$
 $-2 = \sqrt{4}$
 $-2 = 2 \quad \times$
 $(3) = \sqrt{(3)+6}$
 $3 = \sqrt{9}$
 $3 = 3 \quad \checkmark$

x = 3

①
②
d) $\sqrt{x^2 - 5x - 5} = (3)^2$

③ $x^2 - 5x - 5 = 9$
 $-9 \quad -A$
 $x^2 - 5x - 14 = 0$
 $a=1 \quad b=-5 \quad c=-14$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$
 $x = \frac{5 \pm \sqrt{25 + 56}}{2}$
 $x = \frac{5 \pm \sqrt{81}}{2}$
 $x = \frac{5 \pm 9}{2}$

$x = \frac{5+9}{2} \quad x = \frac{5-9}{2}$
 $x = \frac{14}{2} \quad x = \frac{-4}{2}$

$x = 7 \quad x = -2$

④ $\sqrt{(7)^2 - 5(7) - 5} = 3$
 $\sqrt{49 - 35 - 5} = 3$
 $\sqrt{9} = 3$
 $3 = 3 \quad \checkmark$
 $\sqrt{(-2)^2 - 5(-2) - 5} = 3$
 $\sqrt{4 + 10 - 5} = 3$
 $\sqrt{9} = 3$
 $3 = 3 \quad \checkmark$

x = 7 & -2

Don't forget, before you can square both sides, isolate the radical to one side!

Example 2: Solve for x :

$$2\sqrt{x-4} + 7 = 1$$

$$\frac{2\sqrt{x-4}}{2} = \frac{-6}{2} \quad \text{Subtract 7 from both sides of the equation}$$

$$\sqrt{x-4} = -3 \quad \text{Divide both sides of the equation by 2}$$

$$x - 4 = 9 \quad \text{Square both sides of the equation}$$

$$x = 13 \quad \text{Add 4 to both sides of the equation}$$

Check:

$$2\sqrt{13-4} + 7 = 1$$

$$2\sqrt{9} + 7 = 1$$

$$2(3) + 7 = 1$$

$$6 + 7 = 1$$

$$13 \neq 1 \quad \mathbf{X}$$

So, our solution is "No Real Solution".

Some equations have **two** square roots. In these cases rearrange the equation to get one square root on each side of the equation, then square both sides.

Example 3: Solve for x :

$$(3\sqrt{x+3})^2 = \sqrt{2x+13}^2$$

$$9(x+3) = 2x+13 \quad \text{Square both sides of the equation}$$

$$9x + 27 = 2x + 13 \quad \text{Simplify left side by distributing the 9}$$

$$7x + 27 = 13 \quad \text{Subtract } 2x \text{ from both sides of the equation}$$

$$7x = -14 \quad \text{Subtract 27 from both sides of the equation}$$

$$x = -2 \quad \text{Divide both sides of the equation by 7}$$

$$\text{Check: } 3\sqrt{-2+3} = \sqrt{2(-2)+13}$$

$$3\sqrt{1} = \sqrt{-4+13}$$

$$3(1) = \sqrt{9}$$

$$3 = 3 \quad \checkmark$$

So, our solution is $x = -2$

Practice: Solve for x :

$$\textcircled{1} \textcircled{2} \textcircled{3} \textcircled{4} \quad \text{g) } \sqrt{3x-1} = \sqrt{x+5}$$

$$\textcircled{3} \quad \begin{array}{r} 3x-1 = x+5 \\ -x \quad -x \\ +1 \quad +1 \end{array}$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

$$\textcircled{4} \quad \sqrt{3(3)-1} = \sqrt{(3)+5}$$

$$\begin{array}{l} \sqrt{9-1} = \sqrt{8} \\ \sqrt{8} = \sqrt{8} \quad \checkmark \end{array}$$

$$\boxed{x=3}$$

$$\textcircled{1} \quad \text{h) } \sqrt{2x-1} - 2\sqrt{x} = 0$$

$$+2\sqrt{x} \quad +2\sqrt{x}$$

$$\textcircled{2} \quad \sqrt{2x-1} = (2\sqrt{x})^2$$

$$\textcircled{3} \quad \begin{array}{r} 2x-1 = 4x \\ -2x \quad -2x \end{array}$$

$$\frac{-1}{2} = \frac{2x}{2}$$

$$x = -\frac{1}{2}$$

extraneous solution

$$\textcircled{4} \quad \sqrt{2(-\frac{1}{2})-1} - 2\sqrt{(-\frac{1}{2})} = 0$$

$$\sqrt{-2} - 2\sqrt{-\frac{1}{2}} = 0$$

$$i = 0$$

$\boxed{\text{No Solution}}$

Example 4: Solve for x :

$$(x + 3)^2 = \sqrt{3x + 7}$$

$$(x + 3)^2 = 3x + 7 \quad \text{Square both sides of the equation}$$

$$(x+3)(x+3)$$

$$x^2 + 6x + 9 = 3x + 7 \quad \text{FOIL/Box Method the left side}$$

$$x^2 + 3x + 2 = 0 \quad \text{Get everything to one side}$$

$$(x + 2)(x + 1) = 0 \quad \text{Factor the left side of the equation}$$

$$x + 2 = 0 \quad x + 1 = 0 \quad \text{Use the Zero Product Property}$$

$$x = -2 \quad x = -1 \quad \text{Solve each linear equation}$$

$$\text{Check: } -2 + 3 = \sqrt{3(-2) + 7} \quad \text{and} \quad -1 + 3 = \sqrt{3(-1) + 7}$$

$$1 = \sqrt{-6 + 7} \quad 2 = \sqrt{-3 + 7}$$

$$1 = \sqrt{1} \quad 2 = \sqrt{4}$$

$$1 = 1 \quad \checkmark \quad 2 = 2 \quad \checkmark$$

So, our solutions are $x = -2$ and $x = -1$

Practice: Solve for x :

①
②
i) $(x-4)^2 = \sqrt{x+16}$ ~~?~~

③
 $(x-4)(x-4) = x+16$

$$x^2 - 4x - 4x + 16$$

$$x^2 - 8x + 16 = x + 16$$

$$\begin{array}{r} -x \quad -16 \\ -x \quad -16 \end{array}$$

$$x^2 - 9x = 0$$

$$x(x-9) = 0$$

$$x=0 \quad x-9=0$$

extraneous ~~0~~ +9

solⁿ

$$x=9$$

④
 $(0-4) = \sqrt{(0)+16}$

$$-4 = \sqrt{16}$$

$$-4 = 4 \quad \times$$

$$(9-4) = \sqrt{(9)+16}$$

$$5 = \sqrt{25}$$

$$5 = 5 \quad \checkmark$$

$$\boxed{x=9}$$

①
②

ii) $\sqrt{6x+34} = (x+3)^2$ ~~?~~

$$(x+3)(x+3)$$

③

$$6x+34 = x^2+6x+9$$

$$\begin{array}{r} -6x \quad -34 \\ -6x \quad -34 \end{array}$$

$$0 = x^2 - 25$$

$$+25 \quad +25$$

$$\sqrt{25} = \sqrt{x^2}$$

$$x = \pm 5 \rightarrow x=5 \quad x=-5$$

④
 $\sqrt{6(5)+34} = (5)+3$

$$\sqrt{30+34} = 8$$

$$\sqrt{64} = 8$$

$$8 = 8 \quad \checkmark$$

$$\sqrt{6(-5)+34} = (-5)+3$$

$$\sqrt{-30+34} = -2$$

$$\sqrt{4} = -2$$

$$2 = -2 \quad \times$$

$$\boxed{x=5}$$