Polynomials 5 - Solving Polynomial Inequalities

Standards: A-REI.11, F-IF.1, F-IF.4, F-IF.5, F-IF.7c, A-APR.3, A-CED.1

HW#10: #1-4

GLO: #3 Complex Thinker

Math Practice: Reason abstractly and quantitatively

Learning Target:

How can we solve f(x) > g(x) involving polynomials?

Warm Up (split screen)

The graphs of three quadratics functions are shown below. The scale used on each axis is 1-unit.

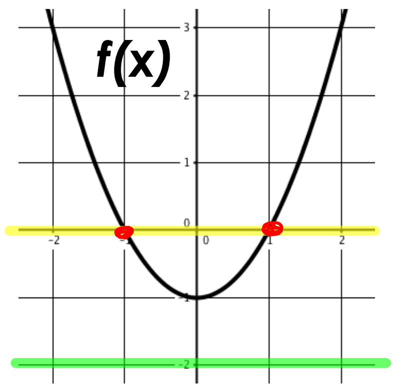
Use the graphs to the right to determine the solution(s) of each of the following equations.

A. $f(x) = 0$

$x = -1$ & 1

B. $g(x) = 0$

$x = -1$ & -3

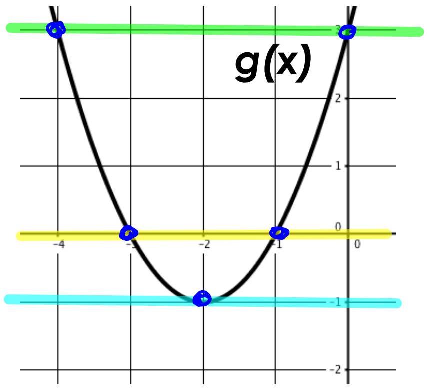


C. $h(x) = 0$

$x = -2$

D. $h(x) = 1$

$x = -1$ & -3



E. $f(x) = -2$

no solution

F. $g(x) = 3$

$x = 0$ & -4

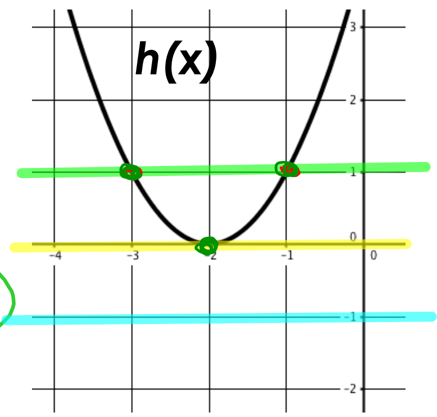
G. $g(x) = -1$

$x = -2$

H. $h(x) = -1$

no solution

(no intersection of h(x) & −1)



Solutions of Polynomial Inequalities

1. The graph of $f(x)=(x+1)(x-2)(x-3)$ is shown

A) Recall that an equation is essentially asking you a question. Write the question that the following equation is asking:

$$f(x)=0$$

"When y=0, what is the x-value?"

B) Use the graph to answer the question you stated above

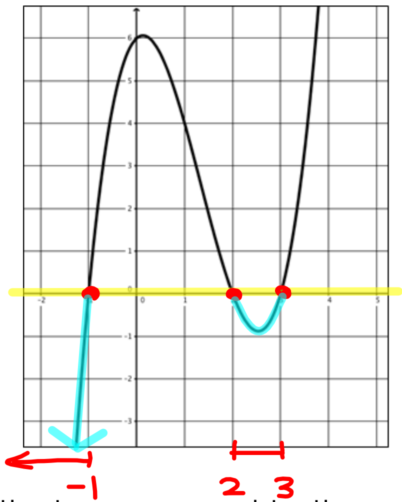
x= -1, 2, 3

C) Write the question that the following inequality is asking:

$$f(x)<0$$

"When y<0, what is the x-value(s)?"

D) Use a colored pen or highlighter to trace over and darken ALL portions of the graph of f(x) that show where $f(x)<0$



E) Shade the intervals along the x-axis that correspond to the portions of the graph that you just traced/darkened.

F) The intervals along the x-axis that you just shaded represent the solution to the inequality (i.e., ALL values of x that satisfy the inequality f(x) < 0). Represent the solution in a complete sentence, set notation, and interval notation.

- All real numbers less than -1 or between 2 & 3.
- $\{x: x<-1 \text{ or } 2<x<3\}$
- $(-\infty,-1)\cup(2,3)$

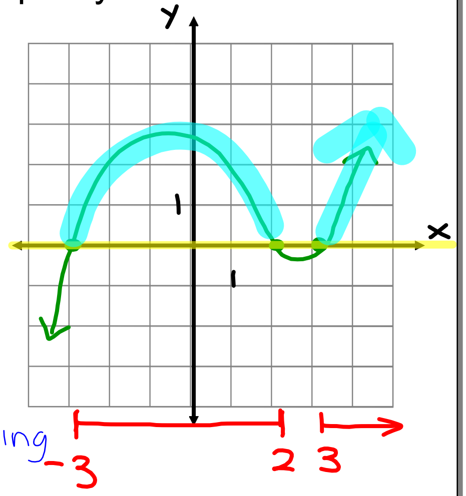
2. Determine the solution to the inequality

$$(x-2)(x+3)(x-3) \geq 0$$

zeros: 2, -3, 3

$x-2=0$
$+2 \quad +2$
$x=2$

eb: ↙ ↗

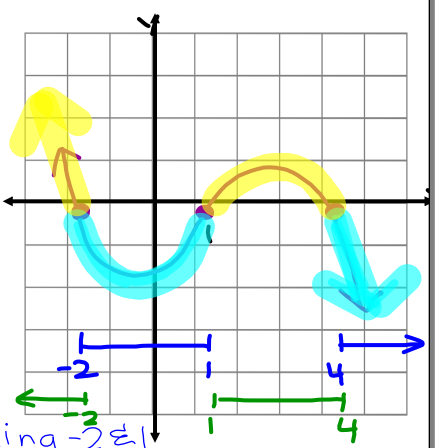


Determine the solution **graphically** and represent the solution in the three standard ways (complete sentence, set notation, interval notation)

- All real numbers between & including -3 & 2 or greater than or equal to 3.
- $\{x: -3 \leq x \leq 2 \text{ or } x \geq 3\}$
- $[-3,2] \cup [3,\infty)$

3. Graph $p(x) = -(x+2)(x-1)(x-4)$
Then use the graph of to determine the solution to the following equations or inequalities. Express the solution to all inequalities in a notation of your choice.

A) $-(x+2)(x-1)(x-4)=0$



zeros: -2, 1, 4

$x+2=0 \quad x-1=0 \quad x-4=0$
$x=-2 \quad x=1 \quad x=4$

EB: ↖ ↘
D = 3
LC = -1

$\boxed{x=-2,1,4}$

B) $-(x+2)(x-1)(x-4) \leq 0$

- All real numbers between & including -2 & 1 or greater than or equal 4.
- $\{x: -2 \leq x \leq 1 \text{ or } x \geq 4\}$
- $[-2,1] \cup [4,\infty)$

C) $-(x+2)(x-1)(x-4) > 0$

- All real numbers less than -2 or between 1 & 4.
- $\{x: x < -2 \text{ or } 1 < x < 4\}$
- $(-\infty, 2) \cup (1,4)$

Comparing the values of two polynomial functions

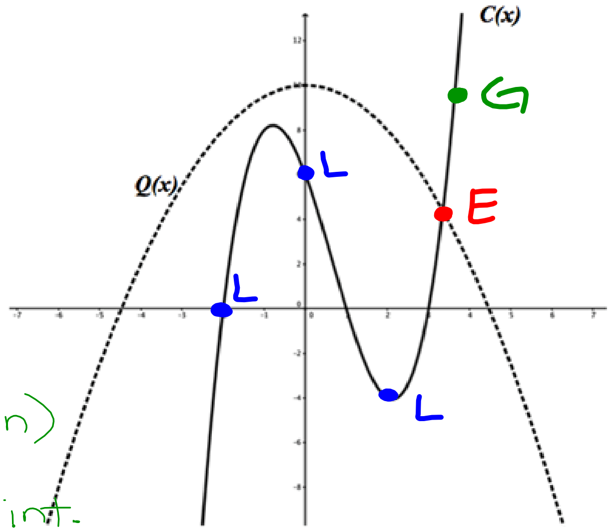
4. The coordinate plane below shows the graph of a cubic function, C(x), and the graph of a quadratic function, Q(x).

A. Mark one point that satisfies the equation C(x) = Q(x). Label this point "E" and justify why this point satisfies the equation C(x) = Q(x).

C(x) & Q(x) intersect at that point

B. Mark one point that satisfies the inequality C(x) > Q(x). Label this point "G" and justify why this point satisfies the equation C(x) > Q(x).

The x-value of C(x) is higher (greater than) the x-value of Q(x) at that point.



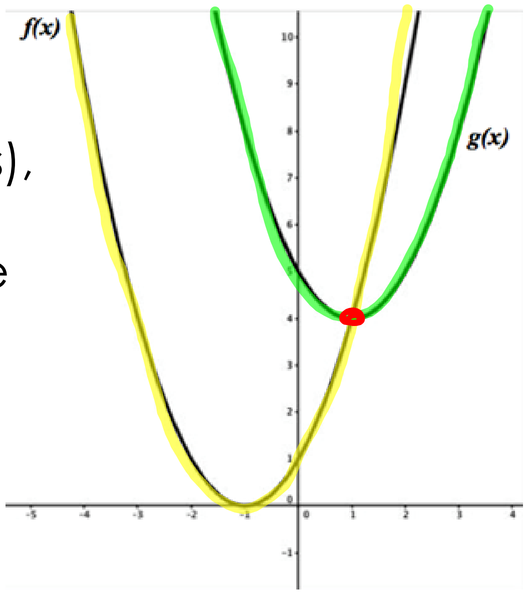
C. Mark one point that satisfies the inequality C(x) < Q(x). Label this point "L" and justify why this point satisfies the equation C(x) < Q(x).

At that point, the x-value of C(x) is less than the x-value of Q(x).

5. The coordinate plane shows the graphs of the following functions:

$$f(x) = x^2 + 2x + 1$$

$$g(x) = x^2 - 2x + 5$$



To find the intersection point(s), we may not want to rely on looking at the graph since the point(s) may be hard to determine exactly.

Remember, we can solve f(x) = g(x) by also using algebraic methods:

$x^2 + 2x + 1 = \cancel{x^2} - \cancel{2x} + \cancel{5}$

$-x^2 \quad +2x \quad -5 \qquad -\cancel{x^2} \quad +\cancel{2x} \quad -5$

$4x - \cancel{4} = 0$

$+\cancel{4} \quad +4$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{4}{4} \qquad x = 1$

Look at the graph.
Do the parabolas seem to intersect at x = __1__?

Check: by substituting x = __1__ into each function.

$f(1) = (1)^2 + 2(1) + 1$

$= 1 + 2 + 1$

$\boxed{f(1) = 4}$

$g(1) = (1)^2 - 2(1) + 5$

$= 1 - 2 + 5$

$= -1 + 5$

$\boxed{g(1) = 4}$

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6. The graph shows a cubic function f and a linear function g.

A) Select all values that appear to be solutions to the equation $f(x) = g(x)$

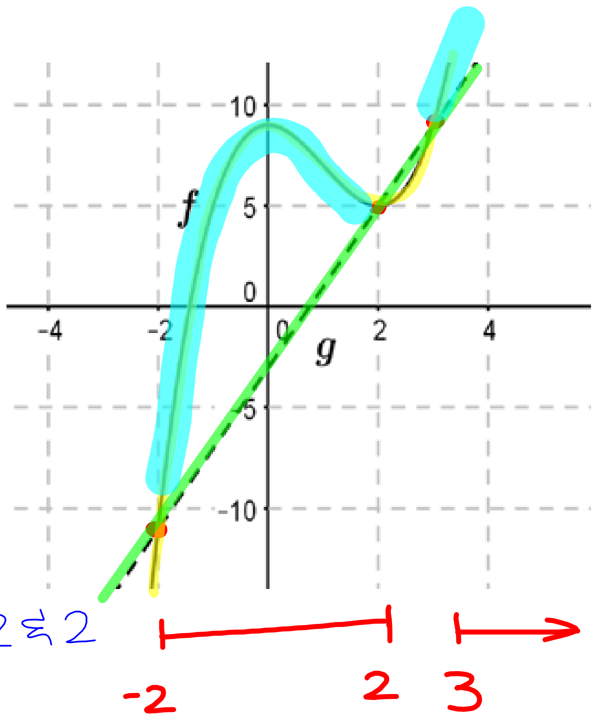
$x = -3$ $x = -2$

$x = -1.5$

$x = 0.75$ $x = 2$

$x = 3$ $x = 5$

B) Shade the parts of the graph to show ALL solutions to the inequlity $f(x) > g(x)$

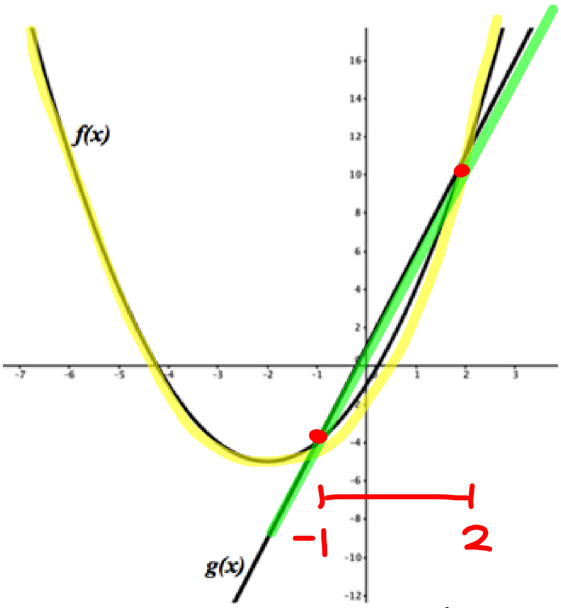


C) Write your answer from part B in any of the three standard forms

- All real numbers between -2 & 2 or greater than 3.
- $\{x: -2 < x < 2 \text{ or } x > 3\}$
- $(-2, 2) \cup (3, \infty)$

7. The graphs of the following functions are shown below:

$$f(x) = x^2 + 4x - 1$$
$$g(x) = 5x + 1$$



A. Solve for all values of x such that

$$f(x) \leq g(x)$$

$x^2 + 4x - 1 \leq 5x + 1$

Write your solution in one of the three standard forms.

- All real numbers between & including -1 & 2.
- $\{x : -1 \leq x \leq 2\}$
- $[-1, 2]$