Exponential Functions 3 - Modeling

Standards: N-Q.2, A-SSE.3c, F-IF.8b, F-LE.2, F-LE.5
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ...exponential functions.

Learning Targets:

-What kinds of real life examples are there for exponential growth & decay?

-What do you need to create exponential models?

Apr 7-10:30 PM

"When are we ever going to use this?!"

Apr 7-10:53 PM

(erase to show)

Exponential Growth Model

$$y = a(1+r)^t$$

a = initial amount

r = rate (decimal)

t = time

(1 + r) → growth factor

Feb 2-8:42 AM

Ex1: In 1995 the cost of tuition at a university was $4300. During the next 8 years The tuition rose 6% each year. Write a model that gives the tuition in *y* dollars *t* years after 1995. How much did the university charge for tuition in 2001?

$y = a(1+r)^t \longrightarrow y = 4300(1+0.06)^t$

$a = 4300$

$r = 6\% = 0.06$

$t = t$

$y = 4300(1.06)^t$

1995 → 2001

$t = 6$ yrs

$y = 4300(1.06)^6$

The tuition in 2001 was $6099.63.

Compound Interest

(erase to show)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = Principal (initial amount)

r = rate (decimal)

t = time (in years)

n = number of times compounded per year

Ex2: You have $1500 to put into a savings account. The bank offers 6% annual interest and 3 different kinds of accounts. Find the balance after 1 year.

$r = 6\% = 0.06$ $P = 1500$ $t = 1\text{yr}$

Acct#1 - compounded yearly

$n = 1$

$A = 1500\left(1 + \frac{0.06}{1}\right)^{1 \cdot 1}$

$A = 1500(1 + 0.06)^{1}$

$A = \$1590$

Acct#2 - compounded semiannually

$n = 2$

$A = 1500\left(1 + \frac{0.06}{2}\right)^{2 \cdot 1}$

$A = 1500(1 + 0.03)^{2}$

$A = \$1591.35$

Acct#3 - compounded quarterly

$n = 4$

$A = 1500\left(1 + \frac{0.06}{4}\right)^{4 \cdot 1}$

$A = 1500(1 + 0.015)^{4}$

$A = \$1592.05$

(erase to show)

Exponential Decay Model

$$y = a(1 - r)^t$$

a = initial amount

r = rate (decimal)

t = time

(1 - r) → decay factor

Feb 2-8:42 AM

Example 3:

You buy a car for $32,000. The value of the car depreciates by 13.25% each year.

decay

a) Write an exponential decay model for the value V of the car after t years.

$a = 32{,}000$

$r = 13.25\% = 0.1325$

$t = t$ yrs

$V = 32000(1 - 0.1325)^t$

$\boxed{V = 32000(0.8675)^t}$

b) What is the value of the car after 5 years?

$t = 5$ yrs $\quad V = 32000(0.8675)^5$

$\boxed{\text{In 5 years, the car is worth } \$15{,}721.62.}$

Sometimes you will be given an equation already created for a scenario, and other times you will have to create your own. Here are more examples..

Apr 20-4:43 PM

1) A leap of leopards is comprised of 20 members. In zoos, leopards normally multiply at a rate of 15% each year.

$a=20$ $r=15\%=0.15$ ↘ growth

a. Write an expression to model the total population of leopards after t-years.

$y=a(1+r)^t \rightarrow y=20(1+0.15)^t$

$y=20(1.15)^t$

b. How many leopards would you expect to find in 18 years?

$t=18$ yrs $y=20(1.15)^{18}$

~~247.5~~ → 247 leopards

c. How long would it take for there to be 100 leopards in the leap? $100=20(1.15)^t$

Guess & Check

$t=9$ yrs	$t=10$ yrs	$t=11$ yrs	$t=12$ years
70	80	93	107

About 12 yrs

OR in a graphing calculator...

$y_1=20(1.15)^x \rightarrow 2^{nd}$ table

2) The value of a new car *t* years after being purchased can be modeled by the equation

$$V = 24900(.89)^t$$

1-r

a. What was the original purchase price of the car?

(a-value) $24900

b. By what percentage does the value of the car depreciate (go down) each year?

(r-value) 11%

$0.89 = 1 - r$

$-1 \quad -1$

$+0.11 = +r$

c. How much will the car be worth after 5 years?

$t = 5$ yrs $\quad V = 24900(0.89)^5$

$\$13,904.31$

d. After how many years will the car be worth half its purchase price? $V = \frac{1}{2}(24900) = \$12,450$

About 6 years

3. The population of a certain microbe t-hours after being placed into a Petri dish is given by

$$P(t) = 1000(2.5)^t$$ growth

a. What was the initial population placed into the dish?

(a-value) 1000 microbes

b. What is the population 1 hour later? $t = 1$

$P = 1000(2.5)^1 \rightarrow$ 2500 microbes

c. If the Petri dish can only safely hold a population of 10,000, approximately how long will it take the population to "run out of room"? $P(t) = 10{,}000$

2 hrs: 6250 | 3 hrs: 15,625 | 2.5 hrs: 9882

About 2.5 hrs

d. By what percentage is the population of this microbe increasing/decreasing?

(r-value)

$1 + r = 2.5$

$-1 \quad -1$

$r = 1.5$

increasing by 150%

4) The U.S. Census recorded the population in the town of Springfield at 124,550 in 2010.

a. In 2009 there were 120,922 people. What percent did the population increase from 2009 to 2010? (Round to the nearest whole percent)

growth

$\frac{\text{new-old}}{\text{old}} \rightarrow \frac{124500-120922}{120922} = 0.0295 = 3\%$

b. Write an expression to model the total population of Springfield after t-years using the rate you found in **a**.

$a = 120{,}922$

$r = 3\% = 0.03$

$t = t \text{ yrs}$

$y = 120{,}922(1+0.03)^t$

$y = 120{,}922(1.03)^t$

c. If the rate of increase were to continue what would be the population in 2020?

$2009 \rightarrow 2020$

$t = 11 \text{ yrs}$

$y = 120{,}922(1.03)^{11}$

The population in 2020 would be 167,384.

5) Your Great-Aunt Millie, whom you've never met, thought education was the most important thing a person could invest in so when she passed away in 2000 she left you $15,000 in a trust you can't touch until you go to college.

compound interest

a. If the trust earns 2.75% interest every year, how much money is in the trust after 4 years? n=1

P= 15,000
r=2.75%=0.0275
t= 4 years
n = 1

$A = 15000\left(1+\frac{0.0275}{1}\right)^{1\cdot 4}$

$A = 15000(1.0275)^4$

$16,719.32

b. If the trust earns 2.75% interest every year, in what year will the money total $20,000?

$20{,}000 = 15000(1.0275)^t$

Guess & check

About 11 years

c. If you start college in 2017 and tuition costs $24,000 will the trust be enough to pay for your first year of tuition?

$A = 15000(1.0275)^t \rightarrow A = 15000(1.0275)^{17}$

2000-2017
t=17

$A = 23{,}789.34$

No, the trust will not be enough to pay for tuition.

d. The executor of the trust told you in 2000 you had the option to split the money into two accounts: $10,000 in account earning 3% annually, and $5000 earning 2.5% annually. Would that earn more or less money?

$10,000 @ 3%

$A = 10000\left(1+\frac{0.03}{1}\right)^{1\cdot 17}$

$A = 10{,}000(1.03)^{17}$

$16,528.48

$5000 @ 2.5%

$A = 5000\left(1+\frac{0.025}{1}\right)^{1\cdot 17}$

$A = 5000(1.025)^{17}$

$7,608.09

$24,136.57

Splitting the money would earn you more.