Polynomials 3b - Graphing Factored Form

Standards: A-APR.3, F-IF.7c

HW#7:
#1-2

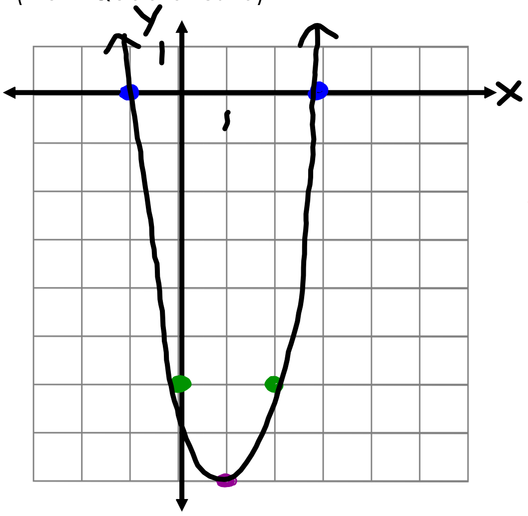
GLO: #4 Quality Producer

Math Practice:#6 Attend to Precision

Learning Target:
What guiding questions should you ask yourself when graphing a polynomial, and how do you answer those questions?

1) Graph $f(x) = 2(x-3)(x+1)$

(From Quadratics2b)



Show how you determined the x-intercepts.

$0 = 2(x-3)(x+1)$

$\cancel{2=0}$ $x-3=0$ $x+1=0$

$x=3$ $x=-1$

Show how you determined the *y*-intercept.

$f(0)=2(0-3)(0+1)$ $(0,-6)$

$=-6$

label your axes & scale

$\frac{3+-1}{2} = \frac{2}{2} = 1$

vertex:

$(1,-8)$

$f(1)=2(1-3)(1+1)$

$=2(-2)(2)$

$=-8$

Graphing Polynomials in Factored Form:

(erase to show)

- Factored form gives us each zero (x-intercept)
  -- Plot these first.
- Notice the multiplicity of each zero
  - > Multiplicity 1 means the graph will **pass through the zero** .
  - > Multiplicity 2 (or any even number) means the graph will **turn around/bounce at the zero** .
  - > Multiplicity 3 (or any odd number) means the graph will **get flat / swerve at the zero** .
- Find the degree and leading coefficient to determine **the end behavior** .
- Find the y-intercept.

Example: Sketch the graph of $k(x) = -(x+4)(x+2)(x-1)^2$

$-1 \quad x \quad x \quad x \quad x$

$-x^4$

Use these guiding questions:

What is the end behavior?

D: 4
LC: −1 ↙ ↘

What are the zeros?

$0 = -(x+4)(x+2)(x-1)(x-1)$

~~$-1=0$~~ $x+4=0$ $x+2=0$ $x-1=0$

$x=-4$ $x=-2$ $x=1$

mult. 2 (bounce)

What is the y-intercept?

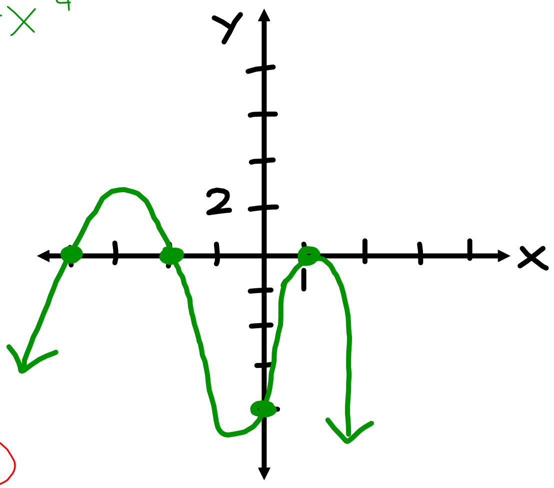
$k(0) = -(0+4)(0+2)(0-1)^2$

$= -(4)(2)(-1)^2$

$= -8(1)$

$= -8$

$(0,-8)$



Practice:

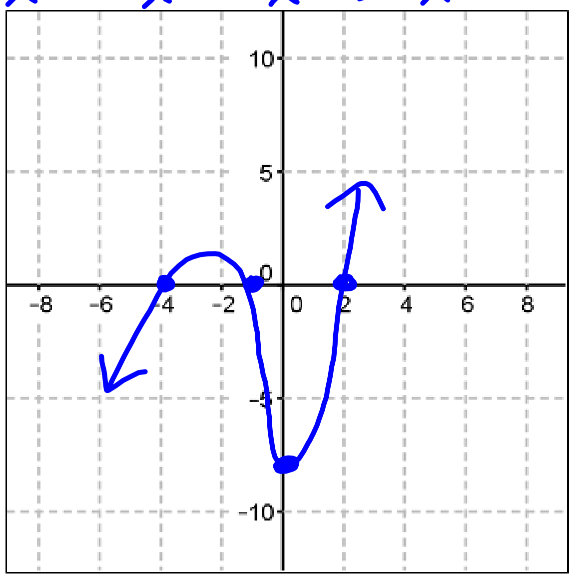
- Use the guiding questions to help with your sketch.
- Your sketch should accurately represent the behavior of the function, however, you do not have to be precise regarding the points where f changes direction.

1. Sketch the graph of **f(x) = (x – 2)(x + 1)(x + 4)**

$x \cdot x \cdot x \rightarrow x^3$

Use these guiding questions:
What is the end behavior?
Explain how you know.

D: 3
LC: 1



What are the zeros?
Explain how you know.

$0 = (x-2)(x+1)(x+4)$

$x-2=0 \quad x+1=0 \quad x+4=0$

$x=2 \quad x=-1 \quad x=-4$

What is the y-intercept?
Explain how you know.

$f(0) = (0-2)(0+1)(0+4)$

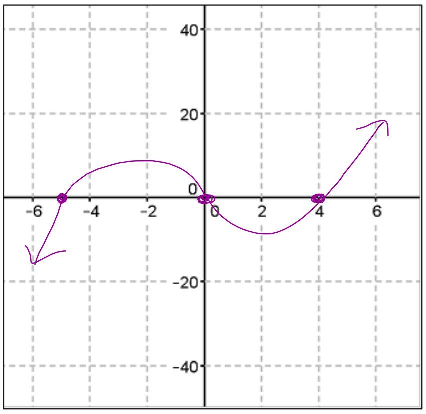
$= (-2)(1)(4)$

$= -8$

$(0,-8)$

2. Sketch the graph of **f(x) = x(x + 5)(x - 4)**

x x x → x^3



What is the end behavior?
Explain how you know.

D: 3
LC: 1
↙ ↗

What are the zeros?
Explain how you know.

$0 = x(x+5)(x-4)$

$x=0$ $x+5=0$ $x-4=0$

$x=-5$ $x=4$

What is the y-intercept?
Explain how you know.

$f(0) = 0(0+5)(0-4)$
$= 0(5)(-4)$
$= 0$

(0,0)

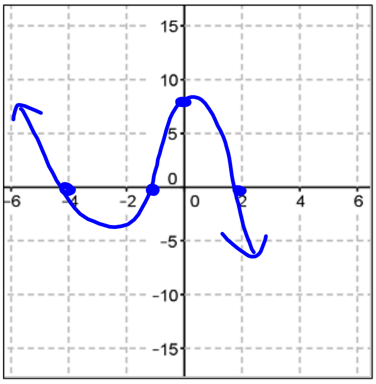
3. Sketch the graph of each function.

f(x) = -(x + 4)(x + 1)(x – 2)

-1 x x x → $-x^3$

↑ ↓ y-int: (0,8)

zeros: -4, -1, 2



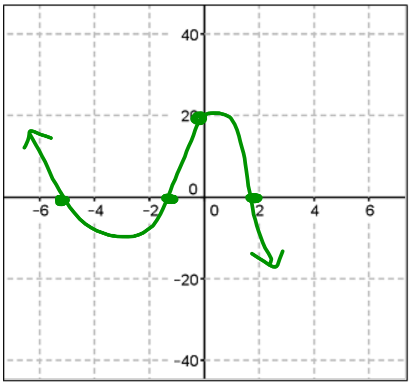
f(x) = -2(x + 5)(x +1)(x – 2)

$-2x^3$

-2 x x x

↑ ↓ y-int: (0,20)

zeros: -5, -1, 2



x x x → x^3

$(x+2)(x-2)(x-2)$

f(x) = (x + 2)(x – 2)²

D: 3
LC: 1
↙ ↗

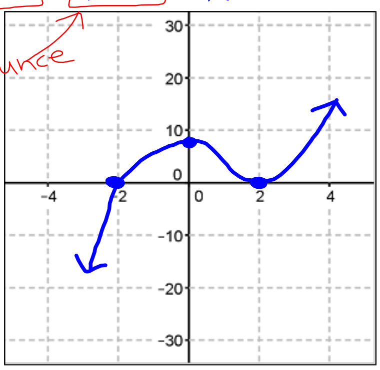
$f(0) = (2)(-2)^2$
$= 2(4)$ (0,8)
$= 8$

$0 = (x+2)(x-2)(x-2)$

$x+2=0$ $x-2=0$ $x-2=0$

$x=-2$ $x=2$ $x=2$

bounce



$-2x^4$

-2 x x x x

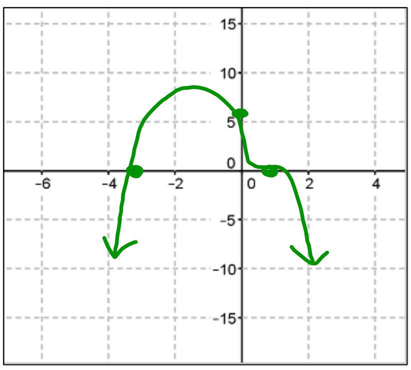
$-2(x-1)(x-1)(x-1)(x+3)$

f(x) = -2(x - 1)³(x +3)

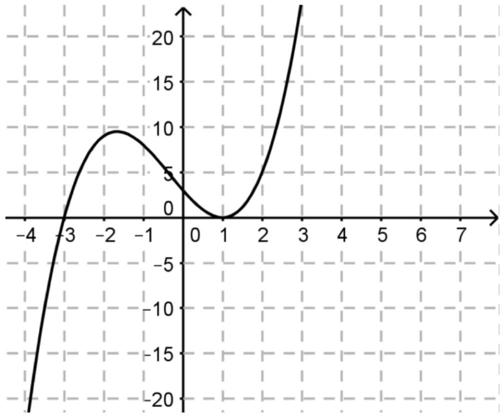
↙ ↘ zeros: 1, -3

swerve

y-int: (0,6)



4. Below is the graph of the function defined by $f(x)=(x-1)^2(x+3)$



a. What is the y-intercept for f?

$f(0)=(0-1)^2(0+3)$

$=$

$(0,3)$

b. What are the x-intercepts for f?

$0=(x-1)(x-1)(x+3)$

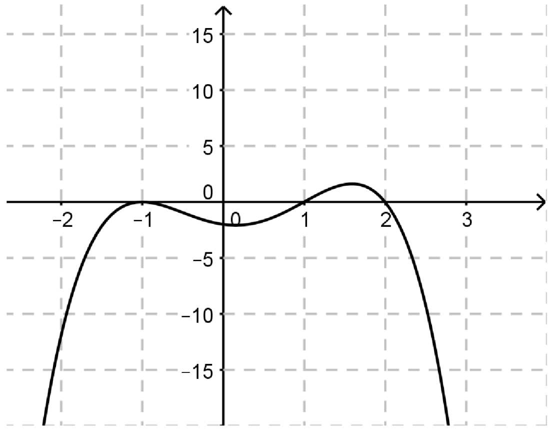
$x=-3, 1 \leftarrow$ bounce

c. Suppose we wish to build a new polynomial function g, such that g has exactly the same intercepts as f, the same end-behavior, and the same behavior close to the x-intercepts, but g has a y-intercept at (0, 15). How could we alter the symbolic representation of f to achieve this? In particular, determine a possible symbolic representation for $g(x)$.

$g(x)=5(x-1)^2(x+3)$

5. Below is the graph of the function defined by

$k(x)=-(x+1)^2(x-1)(x-2)$



a. What is the y-intercept for k?

$k(0)=-(0+1)^2(0-1)(0-2)$

$=-(1)^2(-1)(-2)$

$=-2$ $(0,-2)$

b. What are the x-intercepts for k?

$0=-(x+1)(x+1)(x-1)(x-2)$

$\swarrow$ bounce

$x=-1, 1, 2$

c. Suppose r is a polynomial function with the same x-intercepts as k, the same end-behavior as k, and the same behavior close to its x-intercepts, but with y-intercept at (0, -1). Determine a possible symbolic representation for $r(x)$.

$k(x)=-\frac{1}{2}(x+1)^2(x-1)(x-2)$

If we only want to change the y-intercept of a polynomial function (and keep the same end behavior and x-intercepts) then we have to modify the leading coefficient.

Example: $f(x)=(x+1)^3(x-2)$

x-intercepts: $(-1,0)$ $(2,0)$ y-intercept: $(0,-2)$

Let's change the y-intercept to (0,-8).

$4(x+1)^3(x-2)$

Let's change the y-intercept to (0,-5).

$2.5(x+1)^3(x-2)$