DO NOW

$$f(x) = 14x$$

Find the following:

a) $f(3)$

$= 14(3)$

$\boxed{f(3) = 42}$

b) $f(t)$

$= 14(t)$

$\boxed{f(t) = 14t}$

c) $f(3s)$

$= 14(3s)$

$\boxed{f(3s) = 42s}$

d) $f(2x+5)$

$= 14(2x+5)$

$\boxed{f(2x+5) = 28x+70}$

Functions 5a - Composition of Functions

Standards: F-BF.1, F-BF.1c

GLO: #3 Complex thinker

Math Practice: #7 Look for and make use of structure

Learning Targets:

What does f(g(x)) mean?

How do you do a composition of functions?

Let's look at the following functions:

$$h(x)=\sqrt{2-5x} \qquad k(x)=\frac{1}{3+7x}$$

While these two functions seem a little complicated, we can understand them better if we view each of them as just one function "inside" of another function.

For example, we can view $h(x)=\sqrt{2-5x}$ as the linear function $g(x)=2-5x$ on the inside and the square root function $f(x)=\sqrt{x}$ on the outside.

Thus, h(x) is what you get when you replace the "x" in $f(x)=\sqrt{x}$ with $g(x)=2-5x$. In other words, $h(x)=\sqrt{g(x)}=\sqrt{2-5x}$.

This is an example of

(erase to show) **Composition of Functions**

(where we have **one function *g* inside of another function *f***).

In particular, we see that $h(x)=\sqrt{2-5x}$ is the composition of the function *f* with the function *g*.

(erase to show) There are 2 ways to denote a composition:

$h(x) = f\left(g(x)\right)$ "h of x equals f of g of x" **=** $h = f \circ g$ "h equals f composed with g"

- Notice that even though we *say* the function f first when reading this name, when evaluating f composed with g at x we first evaluate g(x) and then evaluate f at g(x) to get f(g(x)).
- If we want to designate only the name of this function and not its value at x we say that $h = f \circ g$.
- Order is important! $f\left(g(x)\right) \neq g\left(f(x)\right)$ ...most of the time.
- f(g(x)) does NOT mean multiply!! Do <u>NOT</u> do $f(x) \cdot g(x)$

We can create the composition of pretty much any two functions we've come in contact with.

<u>Example 1:</u> Suppose $f(x) = 3x^2$ and $g(x) = 2x - 1$

a) To find f(g(x)) you want to plug *g* into *f*. Start with *f* and replace the x with *g*, that is, replace the x in $3x^2$ with (2x - 1).

$$f\left(g(x)\right) = 3(\underline{\qquad})^2 = 3\left(2x-1\right)^2$$

$3(2x-1)(2x-1)$

$\boxed{f(g(x)) = 12x^2 - 12x + 3} \leftarrow 3(4x^2 - 4x + 1)$

Notice that we were careful to put parentheses around the "g(x)" portion that we are substituting for x. This is frequently necessary when dealing with the composition of functions.

b) What if we asked instead for g(f(x))? What would the expression be? Is it equal to f(g(x))?

$$\underbrace{2(3x^2)}_{6x^2} - 1$$

$6x^2 \quad - 1$

$\boxed{g(f(x)) = 6x^2 - 1}$

For the compositions below, let:

$$k(x) = 2x^2 \qquad h(x) = 3 - 7x \qquad p(x) = \sqrt{x}$$

$$q(x) = \frac{1}{x} \qquad t(x) = |x|$$

Example 2:

a) $k(h(x)) = 18 - 84x + 49x^2$

$2(3-7x)^2$

$(3-7x)(3-7x)$

$2(9 - 42x + 49x^2)$

$18 - 84x + 98x^2$

or

$98x^2 - 84x + 18$

$h(k(x))$

b) $(h \circ k)(x) = 3 - 14x^2$

$3 - 7(2x^2)$

$3 - 14x^2$

$p(h(x))$

c) $(p \circ h)(x) = \sqrt{3-7x}$

$\sqrt{(3-7x)}$

d) $q(k(x)) = \frac{1}{2x^2}$

$\frac{1}{(2x^2)}$

For the compositions below, let:

$k(x) = 2x^2$ $h(x) = 3 - 7x$ $p(x) = \sqrt{x}$

$q(x) = \frac{1}{x}$ $t(x) = |x|$

Example 2:

e) $t(h(x)) = |3-7x|$

$|3-7x|$

f) $(h \circ t)(x) = 3-7|x|$

$h(t(x))$

$3-7(\ |x|\ )$

$3-7|x|$

g) $(h \circ h)(x) = -18+49x$

$h(h(x))$

$3-7(3-7x\ )$

$3-21+49x$

$-18+49x$

h) $k(k(x)) = 8x^4$

$2(2x^2)^2$

$2(4x^4)$

$8x^4$