Functions 4b - Piecewise Functions

Standards: F-IF.7b

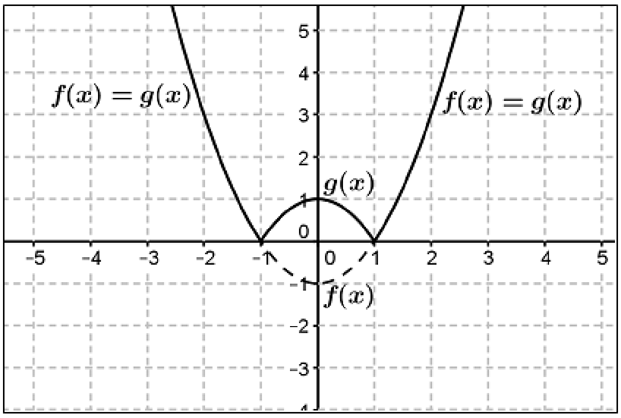
HW#2: Func 4b #1-6

Learning Target:

How do you graph a piecewise function?

In the previous lesson on Absolute Value Functions we saw that for different values of x we sometimes need to define our function using a different symbolic representation.

For example, the solid line graph above represents the function $g(x)=|x^2-1|$. The dotted line represents part of the function defined by *f*(x) = $x^2 - 1$. Since those points on the graph of *f* with positive heights are unaffected by the absolute value, the parts of the graphs for *f* and *g* corresponding to x ≤ 1 and x ≥ 1 are identical.



1. Is there a way to *symbolically* represent that part of *g* lying between x = -1 and x = 1 without using the absolute value symbol? Hint: What is the relationship between the symbolic representation of *f* and that of *g* for these values?

When a different symbolic formula is used for different parts of the domain we say that the function is defined in pieces and we call such a function a **Piecewise Function.**

Example: Let's write the previous absolute value function in terms of its separate pieces. When *f*(x) ≥ 0, i.e. for x ≤ -1 or x ≥ 1, the absolute value does not affect *f*, so for those values of x, $f(x)=x^2-1$

For those values of x where *f*(x) is negative, i.e. for -1 < x < 1, then the absolute value gives the negative of that negative value, which makes it positive. So, for these values *f*(x) is given by $f(x)=-(x^2-1)=1-x^2$

Representing this as a single function we use the notation

$$f(x)=\begin{cases} x^2-1 & x\le -1 \\ 1-x^2 & -1<x<1 \\ x^2-1 & x\ge 1 \end{cases}$$

Piecewise functions come up very naturally in other ways in the real world. For example, if you begin filling up your cylindrical swimming pool with a hose, the height of the water in the pool changes in a linear manner, with the slope dependent on the radius of the pool and the rate at which your hose is delivering the water. If after 3 hours you add a second hose the function that describes the height changes to a different linear function. So, how do we represent the height of the water in the pool as a single function, beginning from the time we started with only one hose until the pool is full?

Answer: Easy, we simply provide the two formulas separately and designate to which part of the domain each formula applies. For example, if our pool takes twenty hours to fill, we may define *h(t) = t* for values of t between 0 and 3, assuming our pool fills one inch per hour using only a single hose, and then *h(t) = 2t +3* for values of t between 3 and 20, assuming the second hose provides an equal amount of water as the first.

We define this more concisely using the following notation:

$$h(t) = \begin{cases} t & , 0 \le t \le 3 \\ 2t + 3 & , 3 < t \le 20 \end{cases}$$

$$h(t)=\begin{cases} t & ,\ 0\le t\le 3 \\ 2t+3 & ,\ 3<t\le 20 \end{cases}$$

2. Fill in the following based on the above definition of h:

a. h(1) = 1 $h(t)=t \Rightarrow h(1)=1$

b. h(4) = 11 $h(t)=2t+3 \Rightarrow h(4)=2(4)+3$

c. h(3) = 3 $h(t)=t \Rightarrow h(3)=3$

Graph the following functions given piecewise on the graph below.

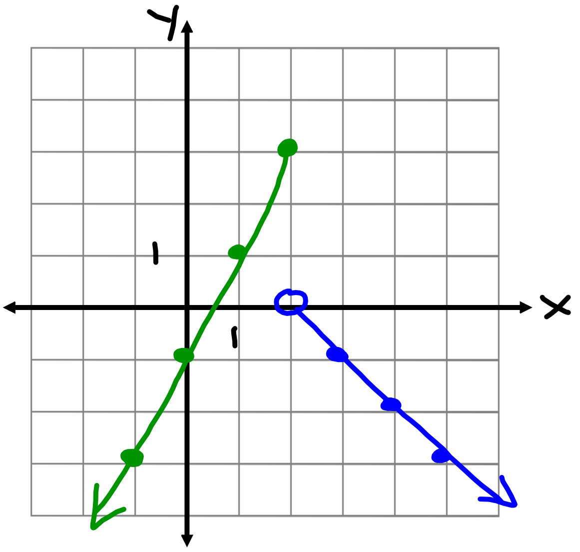
3. $f(x)=\begin{cases} \overset{\text{linear}}{2x-1}, & x\le 2 \\ \overset{\text{linear}}{-x+2}, & x>2 \end{cases}$

$2x-1$

x	y
2	3
1	1
0	-1
-1	-3

$-x+2$

x	y
2	0
3	-1
4	-2
5	-3



What is the range of f? ____________

- All real numbers less than or equal to 3.
- $\{y : y \le 3\}$
- $(-\infty, 3]$

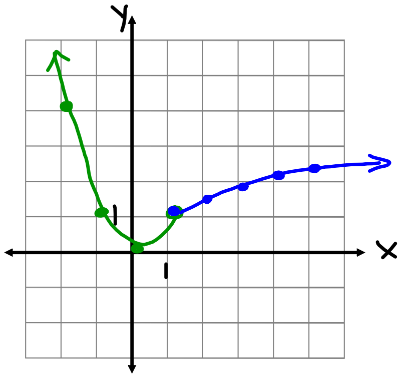
4. $f(x)=\begin{cases} x^2, & x<1 \\ \sqrt{x}, & x\geq 1 \end{cases}$ quadratic / square root

x^2

x	y
1	1
0	0
-1	1
-2	4

$\sqrt{x}$

x	y
1	1
2	1.4
3	1.7
4	2
5	2.2



What is the range of f? ____________

- All real numbers greater than or equal to 0.
- $\{y: y\geq 0\}$
- $[0,\infty)$

5. $f(x)=\begin{cases} \sqrt{x}, & x\geq 0 \\ \dfrac{1}{x} & x<0 \end{cases}$ square root / rational

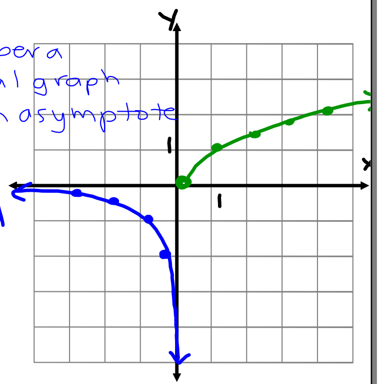
*remember a rational graph has an asymptote at 0.

$\sqrt{x}$

x	y
0	0
1	1
2	1.4
3	1.7
4	2

$\frac{1}{x}$

x	y
0	undefined
-1	-1
-2	-0.5
-3	-0.33
-0.5	-2



What is the range of f? $\mathbb{R}$

6. $f(x)=\begin{cases} 2x, & x<0 \\ 3, & 0\leq x\leq 2 \\ -x^2 & x>2 \end{cases}$ linear / linear / quadratic

$2x$

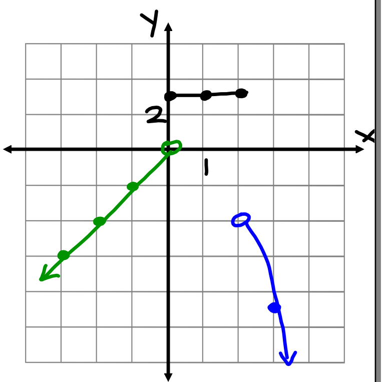
x	y
0	0
-1	-2
-2	-4
-3	-6

3

x	y
0	3
1	3
2	3

$-x^2$

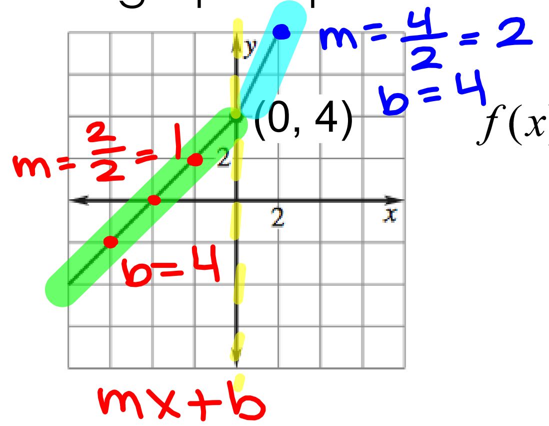
x	y
2	-4
3	-9
4	-16
5	



What is the range of f? ____________

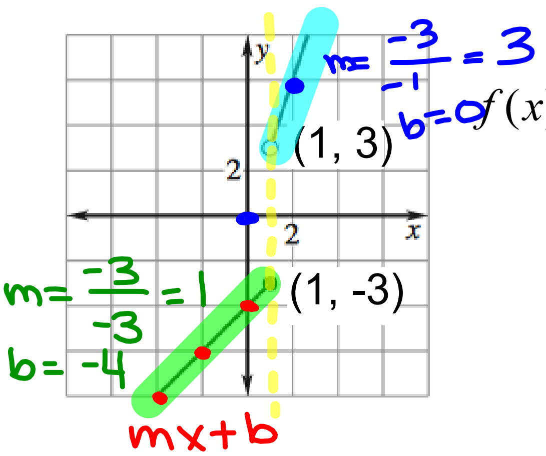
- All real numbers equal to 3 or less than 0.
- $\{y: y=3 \text{ or } y<0\}$
- $(-\infty,0)\cup[3,3]$

7. Given the graph of f below, find its symbolic piecewise representation. Assume the non-linear part of the graph is parabolic.



$$f(x) = \begin{cases} x + 4 & x \leq 0 \\ 2x + 4 & x > 0 \end{cases}$$

8. Given the graph of f below, find its symbolic piecewise representation. Assume the non-linear part of the graph is parabolic.



$$f(x) = \begin{cases} x - 4 & x \leq 1 \\ 3x & x > 1 \end{cases}$$