Quadratics 7 - Solving Quadratic Inequalities

Standards: A-REI.7, A-REI.11, F-IF.7a

HW#2: Quads 7 #1-6

GLO: #3 Complex Thinker

Math Practice: Reason abstractly & Quantitatively

Learning Targets:

How do you write inequality solutions 3 ways?

How do you find the answer to a Quadratic Inequality?

Oct 30-9:18 PM

The number of solutions to equations is usually finite, but the number of solutions to inequalities is usually infinite. Therefore, we must learn how to write our answers. There are **<u>three standard forms</u> for writing solutions to inequalities** and it is important to be able to move between all three. The following example illustrates the three methods:

<u>Example 1</u>: Solve for x such that $2x - \cancel{1} \geq 2$.

$\quad\quad\quad\quad +1 \quad +1$

Solution: Add 1 to both sides $\quad \frac{\cancel{2}x}{\cancel{2}} \geq \frac{3}{2}$

Divide both sides by 2 $\quad$ **$x \geq 3/2$**

Form1:

Complete Sentence

All real numbers greater than or equal to $\frac{3}{2}$.

Form2:

Set Notation $\quad \{x : x \geq \frac{3}{2}\}$

Read this translation as: "The set of all numbers x such that x is greater than or equal to three-halves."

Form3:

Interval notation $\quad \left[\frac{3}{2}, \infty\right)$

$\leq \; \geq \; [\;]$

$< \; > \; (\;)$

(erase to show)

Notice that with interval notation if we want to include an endpoint we use a **<u>square bracket</u>** [ ] on that side of the interval and an **<u>open parenthesis</u>** ( ) if we want a strict inequality. Since *infinity* is a concept and not a number, we always use an open parenthesis for positive or negative infinity.

Nov 18-10:24 PM

Practice #1:

Solve for x such that 5x + 1 < 11. Write your answer using all three standard forms.

- All real numbers less than 2.
- $\{x : x < 2\}$
- $(-\infty, 2)$

$$5x + \cancel{1} < 11$$
$$\quad -1 \quad\;\; -1$$
$$\frac{\cancel{5}x}{\cancel{5}} < \frac{10}{5}$$
$$x < 2$$

It is best to graphically solve inequalities that involve functions other than linear functions. The following example illustrates the problem with solving even the simplest quadratic inequality algebraically.

Example 2: Solve for x such that $x^2 - 4 > 0$.

Attempt at an algebraic solution:

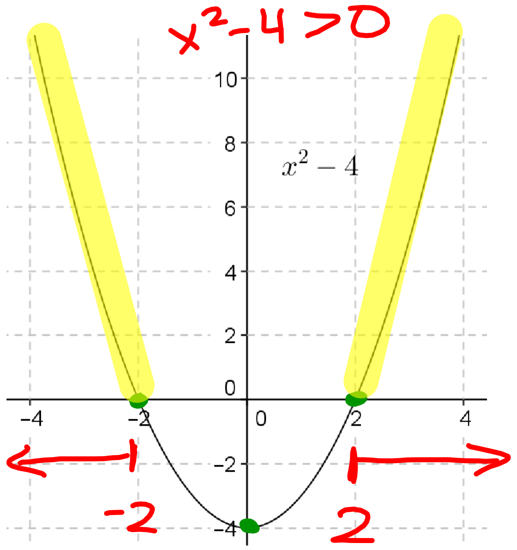
Add 4 to both sides $x^2 > 4$

Take the square root $x > \pm 2$

$x > 2$ $x > -2$

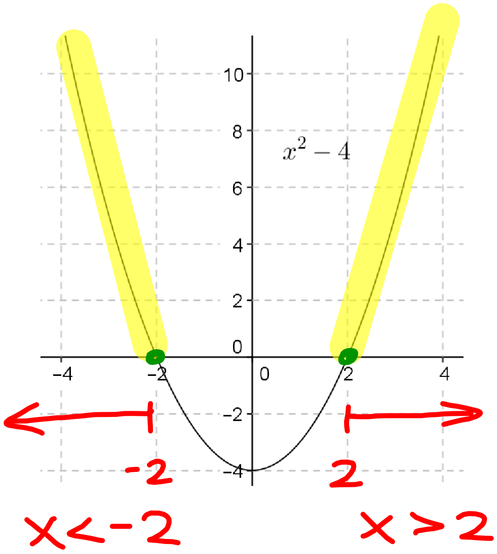
But what does that mean?

But what does this mean, x is greater than plus or minus 2? Is zero such a number? Surely zero is greater than -2. But, if we plug zero into our original inequality we get $0^2 - 4 > 0$, which is the same as $-4 > 0$, which is clearly not true. So, what should we do? Below is the graph of $f(x) = x^2 - 4$. Remember, each point on the graph of f has as its height f(x). So, if we wish to solve $f(x) > 0$ for x, all we need to do is to find all the x values corresponding to points on the graph with positive height. On the graph we see that the height of the point with x-coordinate 0 is negative 4 and hence is not greater than zero. This tells us that 0 is NOT a solution to $f(x) > 0$.



On the other hand, we see that the height of the point corresponding to x = 3, namely (3, 5), is above the x-axis and hence has positive height. Mark this point on the graph. Therefore, x = 3 is one possible solution to the inequality. Checking, we see that $3^2 - 4 = 9 - 4 = 5$, which is indeed greater than 0.

Shade the portion of the graph corresponding to ALL points with positive height. These are precisely those points that lie on the graph and are above the y-axis. But, these points are NOT the solution to the inequality; it is ***the x-coordinates of these points*** that represent the solutions to the inequality. Shade those x-coordinates. What you should find is that each point with x-coordinate less than -2 or greater than 2 has positive height. Therefore, our solution is:



$x^2 - 4 > 0$

Form 1: All real numbers less than -2 or greater than positive 2.

Form 2:

$$\{x : \quad x < -2 \text{ or } x > 2\}$$

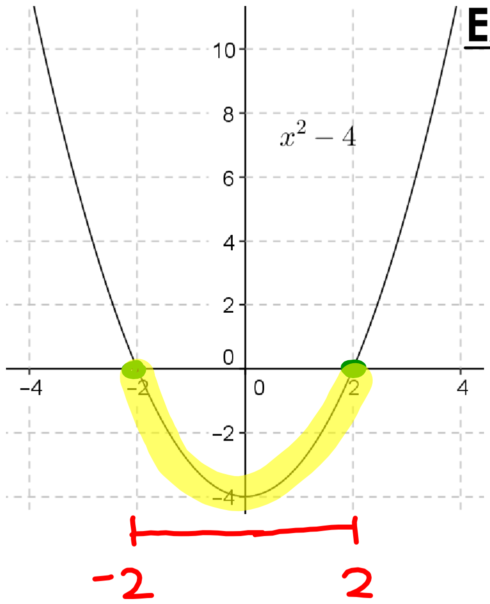
Note: we use "or" instead of "and' since it is impossible for a number x to be both less than -2 *and* greater than positive 2. Notice also that set notation is simply shorthand for the complete sentence, and when read as "The set of all numbers x such that x is less than negative 2 or greater than positive 2," mimics the complete

Form 3:

"or"

$$(-\infty, -2) \cup (2, \infty)$$

Note: we use the **Union** sign for "or" when using interval notation.



$-2 \leq x \leq 2$

Example 3: Solve for x such that

$$x^2 - 4 \leq 0$$

Form : All real numbers... between & including -2 & 2.

Form 2:

{x: $-2 \leq x \leq 2$ }

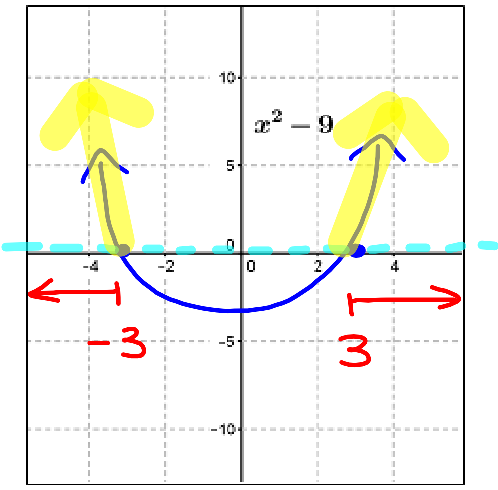
Form 3: [-2 , 2]

Summary note: When we want all numbers between *a* and *b*, where *a* < *b*, we use the notation *a* < x < *b*, whereas when we want all numbers to the left of *a* or to the right of *b* we use the notation x < *a* or x > *b*. It is NOT appropriate to write *b* < x < *a*. This would mean all numbers greater than *b* AND less than *a*. There are no such numbers since *b* > *a*. This is a subtlety involving the difference between the words "and" and "or."

Practice

2. Solve for x such that $x^2 - 9 \geq 0$. Write your answers using the three standard forms.

Hint: first graph $f(x) = x^2 - 9$.



• concave up

$x^2 - \cancel{9} = 0$
$\quad \cancel{+9} \quad +9$
$\sqrt{x^2} = \sqrt{9}$
$x = \pm 3$

• All real numbers less than or equal to -3 or greater than or equal to 3.

• $\{x : x \leq -3 \text{ or } x \geq 3\}$

• $(-\infty, -3] \cup [3, \infty)$

3. Solve for x such that $2x^2 - 7x + 3 < 0$

Write your answers using the three standard forms.

x-int: factoring or quadratic formula

$2x^2 - 7x + 3 = 0$

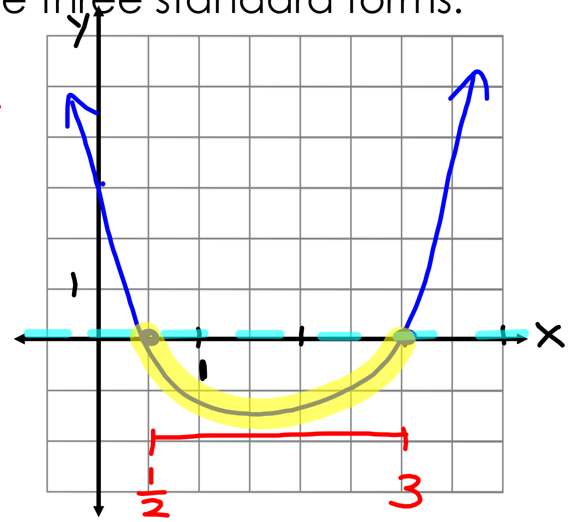
$(2x - 1)(x - 3) = 0$

$2x - 1 = 0 \qquad x - 3 = 0$

$2x = 1 \qquad x = 3$

$x = \frac{1}{2}$

x-int: $(\frac{1}{2}, 0)$ & $(3, 0)$
concave up



$\frac{1}{2} < x < 3$

• All real numbers between $\frac{1}{2}$ & 3.

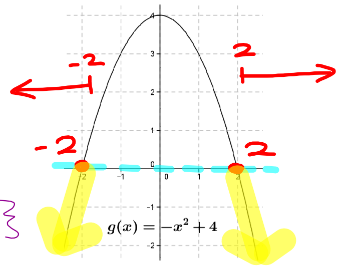
• $\{x : \frac{1}{2} < x < 3\}$

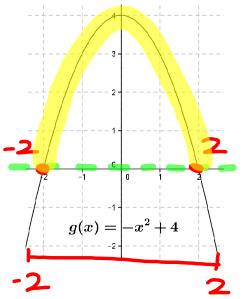
• $(\frac{1}{2}, 3)$

4. Solve the following inequalities graphically (be sure to label and mark/shade the values of x ***on the x-axis*** where the following are true on the graph provided). Write your answers in the three standards forms.

a) $-x^2+4<0$

- All real numbers less than -2 or greater than 2.
- $\{x: x<-2 \text{ or } x>2\}$
- $(-\infty,-2)\cup(2,\infty)$





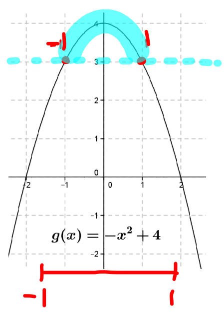
b) $0 \le -x^2+4$

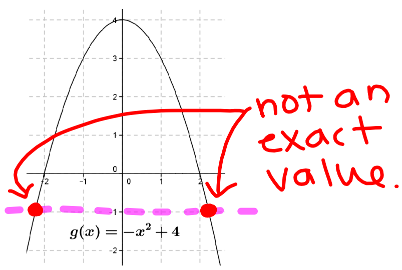
$-x^2+4 \ge 0$

- All real numbers between & including -2 & 2.
- $\{x: -2 \le x \le 2\}$
- $[-2, 2]$

c) $-x^2+4>3$

- All real numbers between -1 & 1.
- $\{x: -1<x<1\}$
- $(-1, 1)$





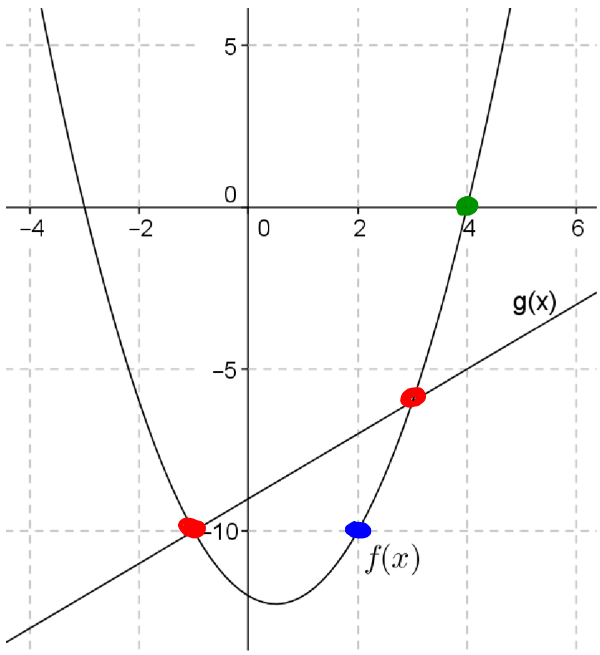
d) $-x^2+4 \le -1$

Reflection: It is not possible to graphically provide an exact answer to problem d above. Explain how you could provide an exact answer combining graphical and algebraic techniques.

Find the intersection of $-x^2+4$ & -1 using a graphing calculator, or by solving the equation $-1=-x^2+4$ (by square rooting or the quadratic formula).

Nov 18-10:51 PM

5. Answer the following based on the graphs of f and g provided.



a) How many solutions are there to the equation f(x) = g(x)?

Two intersections so two solutions

b) How many solutions are there to the equation f(x) < g(x)?

An infinite # of solutions.

c) Mark a point on the graph of g that satisfies f(x) < g(x). Approximate the x-coordinate of this point.

$x \approx 2$

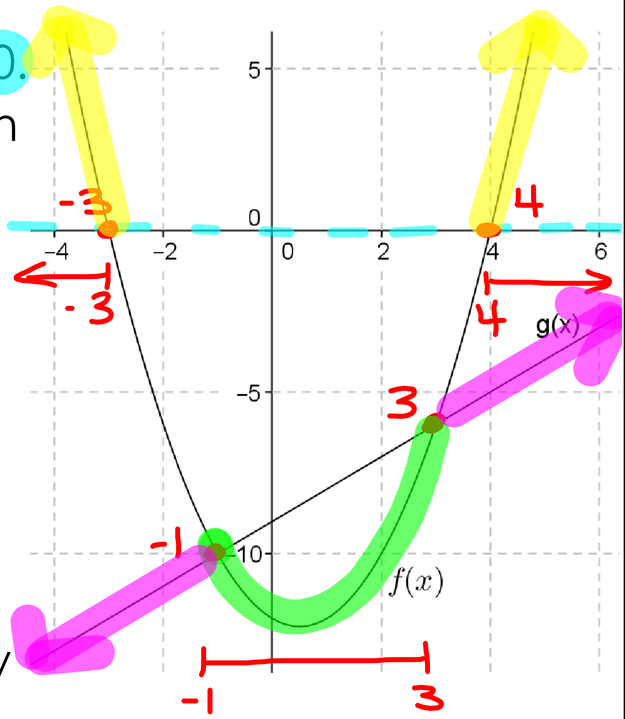
d) Mark a point on the graph of g that satisfies f(x) > g(x). Approximate the x-coordinate of this point.

$x \approx 4$

5. Answer the following based on the graphs of f and g provided.

e) Solve for x such that f(x) > 0. Be sure to write your answer in the three standard forms

- All real numbers less than −3 or greater than 4.
- $\{x: x<-3 \text{ or } x>4\}$
- $(-\infty,-3)\cup(4,\infty)$



f) Solve for x such that f(x)<g(x). Be sure to write your answer in the three standard forms. You may assume that *f(x)* = *g(x)* only at integer values of x.

- All real numbers between −1 & 3.
- $\{x: -1<x<3\}$
- $(-1,3)$

g) Solve for x such that g(x) ≤ f(x). Be sure to write your answer in the three standard forms

- All real numbers less than or equal to −1 or greater than or equal to 3.
- $\{x: x\le -1 \text{ or } x\ge 3\}$
- $(-\infty,-1]\cup[3,\infty)$