Functions 7b - Domains For Functions Defined Symbolically

Standards: **F-IF.5**, A-SSE.3a, A-REI.3, A-REI.4b, A-REI.11, F-IF.4, F-IF.7a

Learning Targets:

What functions have domain restrictions?

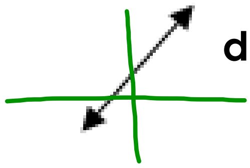
How do you find the domain of those equations?

Part I: Functions with no domain exclusions

Many functions have domains that are defined for each real number x. For such functions, f(x), every real number can be substituted for x in the symbolic representation of f(x). As a result, the graph includes points whose x-values range from $-\infty$ on the left to $+\infty$ on the right. The following parent functions have this property.

Linear $f(x) = x$

ex. $f(x) = 2x - 1$

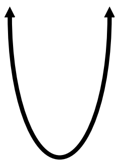


dom(f)= $\mathbb{R}$

$(-\infty, \infty)$

Quadratic $f(x) = x^2$

ex. $f(x) = x^2 - 2x + 1$

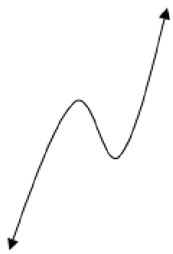


dom(f)= $\mathbb{R}$

$(-\infty, \infty)$

Cubic $f(x) = x^3$

ex.
$f(x) = x^3 - x^2 - 2x + 2$

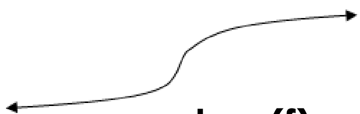


dom(f)= $\mathbb{R}$

$(-\infty, \infty)$

Cube Root $f(x) = \sqrt[3]{x}$

ex. $f(x) = \sqrt[3]{2x - 1}$



dom(f)= $\mathbb{R}$

$(-\infty, \infty)$

Absolute Value $f(x) = |x|$

ex. $f(x) = |2x - 1|$

dom(f)= $\mathbb{R}$

$(-\infty, \infty)$

Part II: Functions with domain exclusions

Some functions have domain exclusions due to restrictions in the operations these functions perform.

Type of Function	Parent Function	Restriction	Domain
Rational	$f(x) = \frac{1}{x}$	Can't divide by 0	dom(f)= $\{x: \quad x \neq 0\}$
Square Root	$f(x) = \sqrt{x}$	Can't take the square root of a negative number	dom(f)= $\{x: \quad x \geq 0\}$

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Additionally, while functions may have domains with no exclusions on their own, such as those from Part I above, exclusions may arise when the parent functions named in part II are composed with them.

For example, the linear function defined by $h(x) = 2x - 1$ has a domain with no exclusions. But when composed with f(x)=1/x, we must exclude any value of x such that h(x)=0 to ensure that we don't attempt to divide by 0.

(can't ÷ by 0)

$$g(x) = \frac{1}{2x-1}$$

$$Dom(g) = \left\{x: \quad 2x - \cancel{1} \neq 0\right\} = \left\{x: \quad x \neq \frac{1}{2}\right\}$$

$+1 \quad +1$

$$\frac{\cancel{2}x}{\cancel{2}} \neq \frac{1}{2}$$

$$x \neq \frac{1}{2}$$

And, when composed with $f(x) = \sqrt{x}$ we must exclude any value(s) of x such that h(x)<0 to ensure that we don't attempt to take the square root of a negative number.

(can't $\sqrt{\ }$ negatives)

$$g(x) = \sqrt{2x-1}$$

$$Dom(g) = \left\{x: \quad 2x - \cancel{1} \geq 0\right\} = \left\{x: \quad x \geq \frac{1}{2}\right\}$$

$+1 \quad +1$

$$\frac{\cancel{2}x}{\cancel{2}} \geq \frac{1}{2}$$

$$x \geq \frac{1}{2}$$

Part III: Finding the domain for functions

composed with $f(x)=\dfrac{1}{x}$

Function: **Restriction:** **Equation or Inequality/ Solution:**

1) $f(x)=\dfrac{1}{x-5}$

can't ÷ by 0

(denominator $\neq 0$)

$x-5\neq 0$

$x\neq 5$

Domain: $\{x : x\neq 5\}$

All real numbers not including 5.

$(-\infty,5)\cup(5,\infty)$

Function: **Restriction:** **Equation or Inequality/ Solution:**

2) $f(x)=\dfrac{1}{2x+7}$

can't ÷ by 0

$2x+7\neq 0$

$-7\quad -7$

$\dfrac{\cancel{2}x}{\cancel{2}}\neq\dfrac{-7}{2}$

$x\neq -\dfrac{7}{2}$

Domain: $\{x : x\neq -\frac{7}{2}\}$

Function:

3) $f(x) = \dfrac{1}{x^2 - 9}$

Restriction:

can't ÷ by 0

Equation or Inequality/ Solution:

$x^2 - 9 \neq 0$

$(x+3)(x-3) \neq 0$

$x+3 \neq 0 \quad x-3 \neq 0$

$x \neq -3 \quad x \neq 3$

Domain: $\{x : x \neq -3, 3\}$

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Function:

4) $f(x) = \dfrac{1}{x^2 - 2x - 24}$

Restriction:

can't ÷ by 0

Equation or Inequality/ Solution:

$x^2 - 2x - 24 \neq 0$

$(x-6)(x+4) \neq 0$

$x-6 \neq 0 \quad x+4 \neq 0$

$x \neq 6 \quad x \neq -4$

Domain: $\{x : x \neq 6, -4\}$

Function: **Restriction:** **Equation or Inequality/ Solution:**

5)

$$f(x) = \frac{1}{x^2 - 10x + 25}$$

can't ÷ by 0

$x^2 - 10x + 25 \neq 0$

$(x-5)(x-5) \neq 0$

$x - 5 \neq 0$

$x \neq 5$

Domain: $\{x : x \neq 5\}$

Function: **Restriction:** **Equation or Inequality/ Solution:**

6)

$$f(x) = \frac{1}{x^2 + 4}$$

can't ÷ by 0

$x^2 + \cancel{4} \neq 0$

$\quad\;\; \cancel{-4} \quad -4$

$\cancel{\sqrt{x^2}} \neq \sqrt{-4}$

no real solution

Domain: $\mathbb{R}$

Part IV: Finding the domain for functions composed with the square root function

Function: **Restriction:** **Equation or Inequality/ Solution:**

7) $f(x) = \sqrt{x+2}$

can't $\sqrt{\ }$ negatives

(radicand ≥ 0)

$x + \cancel{2} \geq 0$

$\quad -\cancel{2} \quad -2$

$x \geq -2$

Domain: $\{x : x \geq -2\}$

Function: **Restriction:** **Equation or Inequality/ Solution:**

8)

$f(x) = \sqrt{3x-1}$

can't $\sqrt{\ }$ negatives

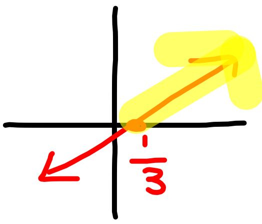
$3x - \cancel{1} \geq 0$

$\quad \cancel{+1} \quad +1$

$\frac{\cancel{3}x}{\cancel{3}} \geq \frac{1}{3}$

$x \geq \frac{1}{3}$

Domain: $\{x : x \geq \frac{1}{3}\}$



Function:

9) $f(x)=\sqrt{x^2-4}$

Restriction:

can't $\sqrt{\ }$ negatives

Equation or Inequality/ Solution:

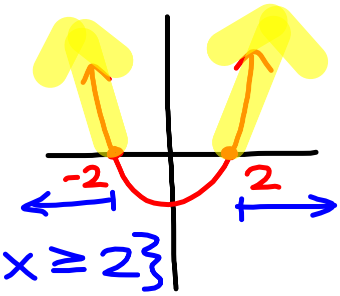
$x^2-\cancel{4}\geq 0$

$\quad\ \ \cancel{+4}\quad +4$

$\cancel{\sqrt{x^2}}\geq\sqrt{4}$

$x\geq\pm 2$

$x\geq 2 \qquad x\geq -2$



Domain:

$\{x: x\leq -2 \text{ or } x\geq 2\}$

$(-\infty,-2]\cup[2,\infty)$

Function:

10) $f(x)=\sqrt{x^2-6x+5}$

Restriction:

can't $\sqrt{\ }$ negatives

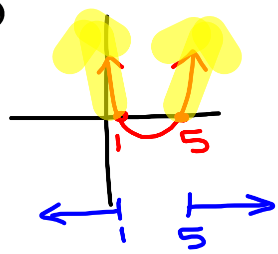
Equation or Inequality/ Solution:

$x^2-6x+5\geq 0$

$(x-5)(x-1)\geq 0$

$x-5\geq 0 \qquad x-1\geq 0$

$x\geq 5 \qquad x\geq 1$



Domain:

$\{x: x\leq 1 \text{ or } x\geq 5\}$

$(-\infty,1]\cup[5,\infty)$

Function:

11) $f(x) = \sqrt{x^2+8x+16}$

Restriction:

can't $\sqrt{\ }$ negatives

Equation or Inequality/ Solution:

$x^2+8x+16 \geq 0$

$(x+4)(x+4) \geq 0$

$x \geq -4$

Domain: $\mathbb{R}$

Function:

12) $f(x) = \sqrt{x^2+1}$

Restriction:

can't $\sqrt{\ }$ negatives

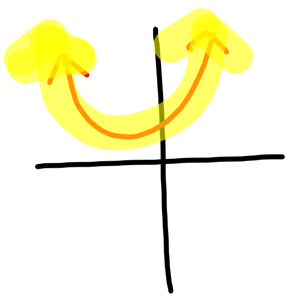
Equation or Inequality/ Solution:

$x^2+\cancel{1} \geq 0$

$\quad\ \ -1 \quad -1$

$\sqrt{x^2} \geq \sqrt{-1}$

no real solution



Domain: $\mathbb{R}$